KATHMANDU UNIVERSITY
End Semester Examination
August/September, 2017

Mark Scored :

AUG 2 3 2017

Level : B. E./B. Sc./B. Tech. | Course : MATH 208
Year : II | Semester : I & II
Exam Roll No. : | Time: 30 mins. | F. M. : 20

Registration No.: | Date :

SECTION "A"
[10 Q. × 1 = 10 marks]

Fill in the blanks space(s) by most appropriate word(s) or symbol(s).

1. If two variables oppose each other then the correlation will be _____

2. A perfect negative correlation is signified by_____

3. The best fitting line is one for which the sum of squares of error is_____

4. The number of patients treated in a dental office on Mondays was recorded for 11 weeks. What are the mean, median, and mode for this set of data?_____
   5, 17, 28, 28, 28, 15, 13, 18, 10, 16, 20

5. If we reject a true null hypothesis we say that _____error has been made.

6. The null hypothesis for an experiment comparing the mean effectiveness of two methods of psychotherapy is _____

7. An investigator quantifies her uncertainty about the estimate of a population mean by reporting $\bar{X} \pm s_{\bar{x}}$. The size of this confidence interval is _____

8. The variance of a binomial distribution with parameters n and p is _____

9. The probability of not rolling any 6's in four rolls of a balanced die is _____

10. The normal distribution is symmetric about_____

SECTION "B"
[10 Q. × 1 = 10 marks]

11. If a random variable X has probability density function

$$f(x) = \begin{cases} \frac{3}{50}\left(x^2 - 4x + 5\right) & 0 \le x \le 5 \\ 0 & x < 0 \text{ or } x > 5 \end{cases}$$

Then, the mode of X is:.....

a. 0 b. 1 c. 2.5 d. 5

12. The mean of a distribution is 14 and the standard deviation is 5. What is the value of the coefficient of variation?_____

a. 60.4% b. 48.3% c. 35.7% d. 27.8%

13. The middle value of an ordered array of numbers is the_____

a. Mode b. Median c. Mean d. Mid Point

14. Which of the following divides a group of data into four subgroups?_____

a. Percentiles b. Deciles c. Median d. Quartiles

15. The number of accidents in a city during 2010 is?_____

a. Discrete random variable b. Continuous random variable
c. Qualitative variable d. Constant

16. A coefficient of correlation is computed to be -0.95 means that_____

a. The relationship between two variables is weak
b. The relationship between two variables is strong and positive
c. The relationship between two variables is strong and negative
d. Correlation coefficient cannot have this value

17. If the function f(x) = 4x represents a probability density function, then which of the following could be the domain of f ?_____

a. $0 < x < \frac{\sqrt{2}}{2}$ b. $0 < x < \sqrt{2}$ c. $-0.5 < x < 0.5$ d. $0 < x < 0.25$

18. A box-and-whisker plot does not show the_____

a. Mean b. Median c. Third quartile d. First quartile

19. Match the following group 1 (charts) with group 2 (use) and select the correct option_____

1. R chart ----------A. study the number of defects per unit
2. C chart ----------B. size of variable is studied
3. P chart ----------C. dispersion of measured data
4. $\bar{X}$ chart ----------D. defective units produced per subgroup

A. 1 – A, 2 – B, 3 – D, 4 - C
B. 1 – C, 2 – D, 3 – B, 4 - A
C. 1 – A, 2 – D, 3 – B, 4 - C
D. 1 – C, 2 – A, 3 – D, 4 – B

20. The probability of not rolling any 4's in four rolls of a balanced die is _____

a. $\frac{625}{1296}$ b. $\frac{1}{1296}$ c. $\frac{1295}{1296}$ d. $\frac{671}{1296}$

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Semester : I & II
F. M. : 55

SECTION "C"
[3Q × 7 = 21 marks]

1. The time it takes to complete a task (in hours) is a random variable X with probability density function. [1+2+2+2]

$$f(x)=\begin{cases} 10cx^2 & 0\le x<0.6 \\ 9c(1-x) & 0.6\le x\le 1.0 \\ 0 & \text{elsewhere} \end{cases}$$

where c is a constant.

i. Find the value of c.
ii. Sketch the graph of this distribution.
iii. Find the probability that the time taken to complete the task will be:
   a more than 0.8 hours
   b between 0.4 and 0.8 hours

2. An agricultural scientist planted alfalfa on several plots of land, identical except for the soil pH. Following are the dry matter yields (in pounds per acre) for each plot. [2 + 2 + 2+1]

pH	Yield
4.6	1056
4.8	1833
5.2	1629
5.4	1852
5.6	1783
5.8	2647
6.0	2131

a. Construct a scatter plot of yield (y) versus pH (x). Verify that a linear model is appropriate.
b. Compute the least squares line for predicting yield from pH.
c. If the pH is increased by 0.1, by how much would you predict the yield to increase or decrease?
d. Predict the yield for a pH of 5.5

3. The thickness, in mm, of metal washers is measured on samples of size 5. The following table presents the means, ranges and standard deviations for 20 consecutive samples. [4+3]

Sample	$\bar{X}$	R	s
1	2.49	0.12	0.07
2	2.45	0.17	0.06
3	2.51	0.13	0.06
4	2.53	0.25	0.09
5	2.49	0.11	0.06
6	2.44	0.11	0.06
7	2.44	0.12	0.05
8	2.42	0.18	0.06
9	2.42	0.08	0.05
10	2.47	0.06	0.02
11	2.54	0.19	0.07
12	2.45	0.09	0.04
13	2.54	0.21	0.07
14	2.55	0.10	0.05
15	2.50	0.25	0.08
16	2.53	0.11	0.04
17	2.58	0.16	0.07
18	2.59	0.09	0.03
19	2.60	0.12	0.05
20	2.56	0.14	0.06

The means are $\bar{\bar{X}} = 2.505, \bar{R} = 0.1395$ and $\bar{s} = 0.057$

a. Calculate the control limits for the R chart. Is the variance under control? If not, delete the samples that are out of control and recomputed $\bar{\bar{X}}$ and $\bar{R}$

b. Based on the sample range R, calculate the control limits for the $\bar{X}$ chart. Based on the control limits is the process mean in control? If not, when is it first detected to be out of control?

SECTION "D"

[6Q. × 4 = 24 marks]

4. Apply the concept of normal distributions to solve the following problems [1+1+2]

a. Find $P(X \leq 21)$ if $X \sim N(15,5)$.

b. Find $P(170 \leq X \leq 190)$ if $X \sim N(175,10)$

c. Find σ^2 if $X \sim N(18, \sigma^2)$ and $P(X \geq 22) = 0.10$

5. Each hour, a 10 m^2 section of fabric is inspected for flaws. The number of flaws observed for the last 20 hours are as follows: [2+ 2]

38 35 35 49 33 48 40 47 45 46
41 53 36 41 51 63 35 58 55 57

a. Compute the upper and lower control limits for c chart.

b. Is the process in control? If not, when is it first detected to be out of control?

6 A 99% confidence interval for a population mean based on a sample of size 64 is computed to be (16.3, 18.7). How large a sample is needed so that a 99% confidence interval will specify mean to within ±1.0?

7. It is thought that a new process for producing a certain chemical may be cheaper than the currently used process. Each process was run 6 times, and the cost of producing 100L of the chemical was determined each time. The results in dollars were as follows:
New Process: 51 52 55 53 54 53
Old process: 50 54 59 56 50 58
Can you conclude that the new process is as consistent as the old process? Can you conclude that the mean cost of the new method is less than that of the old methods? Test at $\alpha = 0.05$ [2+2]

8. A sample of 87 glass sheets has a mean thickness of 4.20mm with a standard deviation of 0.10 mm. [2+2]
   a. Find a 98% confidence interval for the population mean thickness.
   b. What is the level of the confidence interval (4.185, 4.215)?

9. A certain surgical operation on the knee is successful 95% of the time. Twelve people have this operation.
   a. What is the probability that the operation will be successful in all 12 cases?
   b. What is the probability of at least one of the operations being unsuccessful?

SECTION "E"

[5Q.× 2 = 10 marks]

10. Suppose an instant lottery ticket is purchased for Rs 20. The possible prizes are Rs 0, Rs 20, Rs 200, Rs 2000, and Rs 10000. Let Z be the random variable representing the amount won on the ticket, and suppose Z has the following distribution:

Z	0	20	200	2000	10000
P(Z)		0.2	0.05	0.001	0.0001

Determine P(0).

11. For the Sum given in Q10, determine E(Z) and interpret its meaning. How much should you expect to gain or lose on average per ticket?

12. Specifications for an aircraft bolt require that the ultimate tensile strength be at least 18kN. It is known that 10% of the bolts have strengths less than 18.3kN and that 5% of the bolts have strengths greater than 19.76kN. It is also known that the strengths of these bolts are normally distributed. Find the mean and standard deviation of the strengths.

13. For Question no. 12 proportion of the bolts meets the strength specifications?

14. A continuous random variable X has the PDF $f(x) = 2x$ if $0 \leq x \leq 1$. Find the cumulative distribution function (CDF) for X.

Table II Cumulative Standard Normal Distribution

$$\Phi(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-u^2/2}\, du$$

z	0.00	0.01	0.02	0.03	0.04	z
0.0	0.500 00	0.503 99	0.507 98	0.511 97	0.515 95	0.0
0.1	0.539 83	0.543 79	0.547 76	0.551 72	0.555 67	0.1
0.2	0.579 26	0.583 17	0.587 06	0.590 95	0.594 83	0.2
0.3	0.617 91	0.621 72	0.625 51	0.629 30	0.633 07	0.3
0.4	0.655 42	0.659 10	0.662 76	0.666 40	0.670 03	0.4
0.5	0.691 46	0.694 97	0.698 47	0.701 94	0.705 40	0.5
0.6	0.725 75	0.729 07	0.732 37	0.735 65	0.738 91	0.6
0.7	0.758 03	0.761 15	0.764 24	0.767 30	0.770 35	0.7
0.8	0.788 14	0.791 03	0.793 89	0.796 73	0.799 54	0.8
0.9	0.815 94	0.818 59	0.821 21	0.823 81	0.826 39	0.9
1.0	0.841 34	0.843 75	0.846 13	0.848 49	0.850 83	1.0
1.1	0.864 33	0.866 50	0.868 64	0.870 76	0.872 85	1.1
1.2	0.884 93	0.886 86	0.888 77	0.890 65	0.892 51	1.2
1.3	0.903 20	0.904 90	0.906 58	0.908 24	0.909 88	1.3
1.4	0.919 24	0.920 73	0.922 19	0.923 64	0.925 06	1.4
1.5	0.933 19	0.934 48	0.935 74	0.936 99	0.938 22	1.5
1.6	0.945 20	0.946 30	0.947 38	0.948 45	0.949 50	1.6
1.7	0.955 43	0.956 37	0.957 28	0.958 18	0.959 07	1.7
1.8	0.964 07	0.964 85	0.965 62	0.966 37	0.967 11	1.8
1.9	0.971 28	0.971 93	0.972 57	0.973 20	0.973 81	1.9
2.0	0.977 25	0.977 78	0.978 31	0.978 82	0.979 32	2.0
2.1	0.982 14	0.982 57	0.983 00	0.983 41	0.983 82	2.1
2.2	0.986 10	0.986 45	0.986 79	0.987 13	0.987 45	2.2
2.3	0.989 28	0.989 56	0.989 83	0.990 10	0.990 36	2.3
2.4	0.991 80	0.992 02	0.992 24	0.992 45	0.992 66	2.4
2.5	0.993 79	0.993 96	0.994 13	0.994 30	0.994 46	2.5
2.6	0.995 34	0.995 47	0.995 60	0.995 73	0.995 85	2.6
2.7	0.996 53	0.996 64	0.996 74	0.996 83	0.996 93	2.7
2.8	0.997 44	0.997 52	0.997 60	0.997 67	0.997 74	2.8
2.9	0.998 13	0.998 19	0.998 25	0.998 31	0.998 36	2.9
3.0	0.998 65	0.998 69	0.998 74	0.998 78	0.998 82	3.0
3.1	0.999 03	0.999 06	0.999 10	0.999 13	0.999 16	3.1
3.2	0.999 31	0.999 34	0.999 36	0.999 38	0.999 40	3.2
3.3	0.999 52	0.999 53	0.999 55	0.999 57	0.999 58	3.3
3.4	0.999 66	0.999 68	0.999 69	0.999 70	0.999 71	3.4
3.5	0.999 77	0.999 78	0.999 78	0.999 79	0.999 80	3.5
3.6	0.999 84	0.999 85	0.999 85	0.999 86	0.999 86	3.6
3.7	0.999 89	0.999 90	0.999 90	0.999 90	0.999 91	3.7
3.8	0.999 93	0.999 93	0.999 93	0.999 94	0.999 94	3.8
3.9	0.999 95	0.999 95	0.999 96	0.999 96	0.999 96	3.9

(*continues*)

Table II Cumulative Standard Normal Distribution (*continued*)

$$\Phi(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-u^2/2} du$$

z	0.05	0.06	0.07	0.08	0.09	z
0.0	0.519 94	0.523 92	0.527 90	0.531 88	0.535 86	0.0
0.1	0.559 62	0.563 56	0.567 49	0.571 42	0.575 34	0.1
0.2	0.598 71	0.602 57	0.606 42	0.610 26	0.614 09	0.2
0.3	0.636 83	0.640 58	0.644 31	0.648 03	0.651 73	0.3
0.4	0.673 64	0.677 24	0.680 82	0.684 38	0.687 93	0.4
0.5	0.708 84	0.712 26	0.715 66	0.719 04	0.722 40	0.5
0.6	0.742 15	0.745 37	0.748 57	0.751 75	0.754 90	0.6
0.7	0.773 37	0.776 37	0.779 35	0.782 30	0.785 23	0.7
0.8	0.802 34	0.805 10	0.807 85	0.810 57	0.813 27	0.8
0.9	0.828 94	0.831 47	0.833 97	0.836 46	0.838 91	0.9
1.0	0.853 14	0.855 43	0.857 69	0.859 93	0.862 14	1.0
1.1	0.874 93	0.876 97	0.879 00	0.881 00	0.882 97	1.1
1.2	0.894 35	0.896 16	0.897 96	0.899 73	0.901 47	1.2
1.3	0.911 49	0.913 08	0.914 65	0.916 21	0.917 73	1.3
1.4	0.926 47	0.927 85	0.929 22	0.930 56	0.931 89	1.4
1.5	0.939 43	0.940 62	0.941 79	0.942 95	0.944 08	1.5
1.6	0.950 53	0.951 54	0.952 54	0.953 52	0.954 48	1.6
1.7	0.959 94	0.960 80	0.961 64	0.962 46	0.963 27	1.7
1.8	0.967 84	0.968 56	0.969 26	0.969 95	0.970 62	1.8
1.9	0.974 41	0.975 00	0.975 58	0.976 15	0.976 70	1.9
2.0	0.979 82	0.980 30	0.980 77	0.981 24	0.981 69	2.0
2.1	0.984 22	0.984 61	0.985 00	0.985 37	0.985 74	2.1
2.2	0.987 78	0.988 09	0.988 40	0.988 70	0.988 99	2.2
2.3	0.990 61	0.990 86	0.991 11	0.991 34	0.991 58	2.3
2.4	0.992 86	0.993 05	0.993 24	0.993 43	0.993 61	2.4
2.5	0.994 61	0.994 77	0.994 92	0.995 06	0.995 20	2.5
2.6	0.995 98	0.996 09	0.996 21	0.996 32	0.996 43	2.6
2.7	0.997 02	0.997 11	0.997 20	0.997 28	0.997 36	2.7
2.8	0.997 81	0.997 88	0.997 95	0.998 01	0.998 07	2.8
2.9	0.998 41	0.998 46	0.998 51	0.998 56	0.998 61	2.9
3.0	0.998 86	0.998 89	0.998 93	0.998 97	0.999 00	3.0
3.1	0.999 18	0.999 21	0.999 24	0.999 26	0.999 29	3.1
3.2	0.999 42	0.999 44	0.999 46	0.999 48	0.999 50	3.2
3.3	0.999 60	0.999 61	0.999 62	0.999 64	0.999 65	3.3
3.4	0.999 72	0.999 73	0.999 74	0.999 75	0.999 76	3.4
3.5	0.999 81	0.999 81	0.999 82	0.999 83	0.999 83	3.5
3.6	0.999 87	0.999 87	0.999 88	0.999 88	0.999 89	3.6
3.7	0.999 91	0.999 92	0.999 92	0.999 92	0.999 92	3.7
3.8	0.999 94	0.999 94	0.999 95	0.999 95	0.999 95	3.8
3.9	0.999 96	0.999 96	0.999 96	0.999 97	0.999 97	3.9

Table III Percentage Points of the χ^2 Distribution[a]

ν \ α	0.995	0.990	0.975	0.950	0.900	0.500	0.100	0.050	0.025	0.010	0.005
1	0.00+	0.00+	0.00+	0.00+	0.02	0.45	2.71	3.84	5.02	6.63	7.88
2	0.01	0.02	0.05	0.10	0.21	1.39	4.61	5.99	7.38	9.21	10.60
3	0.07	0.11	0.22	0.35	0.58	2.37	6.25	7.81	9.35	11.34	12.84
4	0.21	0.30	0.48	0.71	1.06	3.36	7.78	9.49	11.14	13.28	14.86
5	0.41	0.55	0.83	1.15	1.61	4.35	9.24	11.07	12.83	15.09	16.75
6	0.68	0.87	1.24	1.64	2.20	5.35	10.65	12.59	14.45	16.81	18.55
7	0.99	1.24	1.69	2.17	2.83	6.35	12.02	14.07	16.01	18.48	20.28
8	1.34	1.65	2.18	2.73	3.49	7.34	13.36	15.51	17.53	20.09	21.96
9	1.73	2.09	2.70	3.33	4.17	8.34	14.68	16.92	19.02	21.67	23.59
10	2.16	2.56	3.25	3.94	4.87	9.34	15.99	18.31	20.48	23.21	25.19
11	2.60	3.05	3.82	4.57	5.58	10.34	17.28	19.68	21.92	24.72	26.76
12	3.07	3.57	4.40	5.23	6.30	11.34	18.55	21.03	23.34	26.22	28.30
13	3.57	4.11	5.01	5.89	7.04	12.34	19.81	22.36	24.74	27.69	29.82
14	4.07	4.66	5.63	6.57	7.79	13.34	21.06	23.68	26.12	29.14	31.32
15	4.60	5.23	6.27	7.26	8.55	14.34	22.31	25.00	27.49	30.58	32.80
16	5.14	5.81	6.91	7.96	9.31	15.34	23.54	26.30	28.85	32.00	34.27
17	5.70	6.41	7.56	8.67	10.09	16.34	24.77	27.59	30.19	33.41	35.72
18	6.26	7.01	8.23	9.39	10.87	17.34	25.99	28.87	31.53	34.81	37.16
19	6.84	7.63	8.91	10.12	11.65	18.34	27.20	30.14	32.85	36.19	38.58
20	7.43	8.26	9.59	10.85	12.44	19.34	28.41	31.41	34.17	37.57	40.00
21	8.03	8.90	10.28	11.59	13.24	20.34	29.62	32.67	35.48	38.93	41.40
22	8.64	9.54	10.98	12.34	14.04	21.34	30.81	33.92	36.78	40.29	42.80
23	9.26	10.20	11.69	13.09	14.85	22.34	32.01	35.17	38.08	41.64	44.18
24	9.89	10.86	12.40	13.85	15.66	23.34	33.20	36.42	39.36	42.98	45.56
25	10.52	11.52	13.12	14.61	16.47	24.34	34.28	37.65	40.65	44.31	46.93
26	11.16	12.20	13.84	15.38	17.29	25.34	35.56	38.89	41.92	45.64	48.29
27	11.81	12.88	14.57	16.15	18.11	26.34	36.74	40.11	43.19	46.96	49.65
28	12.46	13.57	15.31	16.93	18.94	27.34	37.92	41.34	44.46	48.28	50.99
29	13.12	14.26	16.05	17.71	19.77	28.34	39.09	42.56	45.72	49.59	52.34
30	13.79	14.95	16.79	18.49	20.60	29.34	40.26	43.77	46.98	50.89	53.67
40	20.71	22.16	24.43	26.51	29.05	39.34	51.81	55.76	59.34	63.69	66.77
50	27.99	29.71	32.36	34.76	37.69	49.33	63.17	67.50	71.42	76.15	79.49
60	35.53	37.48	40.48	43.19	46.46	59.33	74.40	79.08	83.30	88.38	91.95
70	43.28	45.44	48.76	51.74	55.33	69.33	85.53	90.53	95.02	100.42	104.22
80	51.17	53.54	57.15	60.39	64.28	79.33	96.58	101.88	106.63	112.33	116.32
90	59.20	61.75	65.65	69.13	73.29	89.33	107.57	113.14	118.14	124.12	128.30
100	67.33	70.06	74.22	77.93	82.36	99.33	118.50	124.34	129.56	135.81	140.17

[a] ν = degrees of freedom.

ible IV Percentage Points of the t Distribution

α	0.40	0.25	0.10	0.05	0.025	0.01	0.005	0.0025	0.001	0.0005
1	0.325	1.000	3.078	6.314	12.706	31.821	63.657	127.32	318.31	636.62
2	0.289	0.816	1.886	2.920	4.303	6.965	9.925	14.089	23.326	31.598
3	0.277	0.765	1.638	2.353	3.182	4.541	5.841	7.453	10.213	12.924
4	0.271	0.741	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	0.267	0.727	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869
6	0.265	0.718	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	0.263	0.711	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8	0.262	0.706	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	0.261	0.703	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
0	0.260	0.700	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
1	0.260	0.697	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
2	0.259	0.695	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
3	0.259	0.694	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
4	0.258	0.692	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140
5	0.258	0.691	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
6	0.258	0.690	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
7	0.257	0.689	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965
8	0.257	0.688	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
9	0.257	0.688	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
0	0.257	0.687	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
1	0.257	0.686	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
2	0.256	0.686	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792
3	0.256	0.685	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.767
4	0.256	0.685	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745
5	0.256	0.684	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725
6	0.256	0.684	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707
7	0.256	0.684	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.690
8	0.256	0.683	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674
9	0.256	0.683	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.659
0	0.256	0.683	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646
0	0.255	0.681	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551
0	0.254	0.679	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460
0	0.254	0.677	1.289	1.658	1.980	2.358	2.617	2.860	3.160	3.373
∞	0.253	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

urce: This table is adapted from *Biometrika Tables for Statisticians*, Vol. 1, 3rd edition, 1966, by permission of the ometrika Trustees.

Table V Percentage Points of the F Distribution

$F_{0.25,\nu_1,\nu_2}$

ν_2 \ ν_1	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	5.83	7.50	8.20	8.58	8.82	8.98	9.10	9.19	9.26	9.32	9.41	9.49	9.58	9.63	9.67	9.71	9.76	9.80	9.85
2	2.57	3.00	3.15	3.23	3.28	3.31	3.34	3.35	3.37	3.38	3.39	3.41	3.43	3.43	3.44	3.45	3.46	3.47	3.48
3	2.02	2.28	2.36	2.39	2.41	2.42	2.43	2.44	2.44	2.44	2.45	2.46	2.46	2.46	2.47	2.47	2.47	2.47	2.47
4	1.81	2.00	2.05	2.06	2.07	2.08	2.08	2.08	2.08	2.08	2.08	2.08	2.08	2.08	2.08	2.08	2.08	2.08	2.08
5	1.69	1.85	1.88	1.89	1.89	1.89	1.89	1.89	1.89	1.89	1.89	1.89	1.88	1.88	1.88	1.88	1.87	1.87	1.87
6	1.62	1.76	1.78	1.79	1.79	1.78	1.78	1.78	1.77	1.77	1.77	1.76	1.76	1.75	1.75	1.75	1.74	1.74	1.74
7	1.57	1.70	1.72	1.72	1.71	1.71	1.70	1.70	1.70	1.69	1.68	1.68	1.67	1.67	1.66	1.66	1.65	1.65	1.65
8	1.54	1.66	1.67	1.66	1.66	1.65	1.64	1.64	1.63	1.63	1.62	1.62	1.61	1.60	1.60	1.59	1.59	1.58	1.58
9	1.51	1.62	1.63	1.63	1.62	1.61	1.60	1.60	1.59	1.59	1.58	1.57	1.56	1.56	1.55	1.54	1.54	1.53	1.53
10	1.49	1.60	1.60	1.59	1.59	1.58	1.57	1.56	1.56	1.55	1.54	1.53	1.52	1.52	1.51	1.51	1.50	1.49	1.48
11	1.47	1.58	1.58	1.57	1.56	1.55	1.54	1.53	1.53	1.52	1.51	1.50	1.49	1.49	1.48	1.47	1.47	1.46	1.45
12	1.46	1.56	1.56	1.55	1.54	1.53	1.52	1.51	1.51	1.50	1.49	1.48	1.47	1.46	1.45	1.45	1.44	1.43	1.42
13	1.45	1.55	1.55	1.53	1.52	1.51	1.50	1.49	1.49	1.48	1.47	1.46	1.45	1.44	1.43	1.42	1.42	1.41	1.40
14	1.44	1.53	1.53	1.52	1.51	1.50	1.49	1.48	1.47	1.46	1.45	1.44	1.43	1.42	1.41	1.41	1.40	1.39	1.38
15	1.43	1.52	1.52	1.51	1.49	1.48	1.47	1.46	1.46	1.45	1.44	1.43	1.41	1.41	1.40	1.39	1.38	1.37	1.36
16	1.42	1.51	1.51	1.50	1.48	1.47	1.46	1.45	1.44	1.44	1.43	1.41	1.40	1.39	1.38	1.37	1.36	1.35	1.34
17	1.42	1.51	1.50	1.49	1.47	1.46	1.45	1.44	1.43	1.43	1.41	1.40	1.39	1.38	1.37	1.36	1.35	1.34	1.33
18	1.41	1.50	1.49	1.48	1.46	1.45	1.44	1.43	1.42	1.42	1.40	1.39	1.38	1.37	1.36	1.35	1.34	1.33	1.32
19	1.41	1.49	1.49	1.47	1.46	1.44	1.43	1.42	1.41	1.41	1.40	1.38	1.37	1.36	1.35	1.34	1.33	1.32	1.30
20	1.40	1.49	1.48	1.47	1.45	1.44	1.43	1.42	1.41	1.40	1.39	1.37	1.36	1.35	1.34	1.33	1.32	1.31	1.29
21	1.40	1.48	1.48	1.46	1.44	1.43	1.42	1.41	1.40	1.39	1.38	1.37	1.35	1.34	1.33	1.32	1.31	1.30	1.28
22	1.40	1.48	1.47	1.45	1.44	1.42	1.41	1.40	1.39	1.39	1.37	1.36	1.34	1.33	1.32	1.31	1.30	1.29	1.28
23	1.39	1.47	1.47	1.45	1.43	1.42	1.41	1.40	1.39	1.38	1.37	1.35	1.34	1.33	1.32	1.31	1.30	1.28	1.27
24	1.39	1.47	1.46	1.44	1.43	1.41	1.40	1.39	1.38	1.38	1.36	1.35	1.33	1.32	1.31	1.30	1.29	1.28	1.26
25	1.39	1.46	1.45	1.44	1.42	1.41	1.40	1.39	1.38	1.37	1.36	1.34	1.33	1.32	1.31	1.29	1.28	1.27	1.25
26	1.38	1.46	1.45	1.43	1.42	1.41	1.39	1.38	1.37	1.37	1.35	1.34	1.32	1.31	1.30	1.29	1.28	1.26	1.25
27	1.38	1.46	1.45	1.43	1.41	1.40	1.39	1.38	1.37	1.36	1.35	1.33	1.32	1.31	1.30	1.28	1.27	1.26	1.24
28	1.38	1.45	1.45	1.43	1.41	1.40	1.39	1.38	1.37	1.36	1.34	1.33	1.31	1.30	1.29	1.28	1.27	1.25	1.24
29	1.38	1.45	1.44	1.42	1.41	1.39	1.38	1.37	1.36	1.35	1.34	1.32	1.31	1.30	1.29	1.27	1.26	1.25	1.23
30	1.36	1.44	1.42	1.40	1.39	1.37	1.36	1.35	1.34	1.33	1.31	1.30	1.28	1.26	1.25	1.24	1.22	1.21	1.19
40	1.35	1.42	1.41	1.38	1.37	1.35	1.33	1.32	1.31	1.30	1.29	1.27	1.25	1.24	1.22	1.21	1.19	1.17	1.15
60	1.34	1.40	1.39	1.37	1.35	1.33	1.31	1.30	1.29	1.28	1.26	1.24	1.22	1.21	1.19	1.18	1.16	1.13	1.10
120	1.32	1.39	1.37	1.35	1.33	1.31	1.29	1.28	1.27	1.25	1.24	1.22	1.19	1.18	1.16	1.14	1.12	1.08	1.00
∞																			

Degrees of freedom for the numerator (ν_1)

Degrees of freedom for the denominator (ν_2)

Source: Adapted with permission from *Biometrika Tables for Statisticians*, Vol. 1, 3rd edition, by E. S. Pearson and H. O. Hartley, Cambridge University Press, Cambridge, 1966.

(*continues*)

Table V Percentage Points of the F Distribution (*continued*)

$F_{0.10,\nu_1,\nu_2}$

ν_2 \ ν_1	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	39.86	49.50	53.59	55.83	57.24	58.20	58.91	59.44	59.86	60.19	60.71	61.22	61.74	62.00	62.26	62.53	62.79	63.06	63.33
2	8.53	9.00	9.16	9.24	9.29	9.33	9.35	9.37	9.38	9.39	9.41	9.42	9.44	9.45	9.46	9.47	9.47	9.48	9.49
3	5.54	5.46	5.39	5.34	5.31	5.28	5.27	5.25	5.24	5.23	5.22	5.20	5.18	5.18	5.17	5.16	5.15	5.14	5.13
4	4.54	4.32	4.19	4.11	4.05	4.01	3.98	3.95	3.94	3.92	3.90	3.87	3.84	3.83	3.82	3.80	3.79	3.78	3.76
5	4.06	3.78	3.62	3.52	3.45	3.40	3.37	3.34	3.32	3.30	3.27	3.24	3.21	3.19	3.17	3.16	3.14	3.12	3.10
6	3.78	3.46	3.29	3.18	3.11	3.05	3.01	2.98	2.96	2.94	2.90	2.87	2.84	2.82	2.80	2.78	2.76	2.74	2.72
7	3.59	3.26	3.07	2.96	2.88	2.83	2.78	2.75	2.72	2.70	2.67	2.63	2.59	2.58	2.56	2.54	2.51	2.49	2.47
8	3.46	3.11	2.92	2.81	2.73	2.67	2.62	2.59	2.56	2.54	2.50	2.46	2.42	2.40	2.38	2.36	2.34	2.32	2.29
9	3.36	3.01	2.81	2.69	2.61	2.55	2.51	2.47	2.44	2.42	2.38	2.34	2.30	2.28	2.25	2.23	2.21	2.18	2.16
10	3.29	2.92	2.73	2.61	2.52	2.46	2.41	2.38	2.35	2.32	2.28	2.24	2.20	2.18	2.16	2.13	2.11	2.08	2.06
11	3.23	2.86	2.66	2.54	2.45	2.39	2.34	2.30	2.27	2.25	2.21	2.17	2.12	2.10	2.08	2.05	2.03	2.00	1.97
12	3.18	2.81	2.61	2.48	2.39	2.33	2.28	2.24	2.21	2.19	2.15	2.10	2.06	2.04	2.01	1.99	1.96	1.93	1.90
13	3.14	2.76	2.56	2.43	2.35	2.28	2.23	2.20	2.16	2.14	2.10	2.05	2.01	1.98	1.96	1.93	1.90	1.88	1.85
14	3.10	2.73	2.52	2.39	2.31	2.24	2.19	2.15	2.12	2.10	2.05	2.01	1.96	1.94	1.91	1.89	1.86	1.83	1.80
15	3.07	2.70	2.49	2.36	2.27	2.21	2.16	2.12	2.09	2.06	2.02	1.97	1.92	1.90	1.87	1.85	1.82	1.79	1.76
16	3.05	2.67	2.46	2.33	2.24	2.18	2.13	2.09	2.06	2.03	1.99	1.94	1.89	1.87	1.84	1.81	1.78	1.75	1.72
17	3.03	2.64	2.44	2.31	2.22	2.15	2.10	2.06	2.03	2.00	1.96	1.91	1.86	1.84	1.81	1.78	1.75	1.72	1.69
18	3.01	2.62	2.42	2.29	2.20	2.13	2.08	2.04	2.00	1.98	1.93	1.89	1.84	1.81	1.78	1.75	1.72	1.69	1.66
19	2.99	2.61	2.40	2.27	2.18	2.11	2.06	2.02	1.98	1.96	1.91	1.86	1.81	1.79	1.76	1.73	1.70	1.67	1.63
20	2.97	2.59	2.38	2.25	2.16	2.09	2.04	2.00	1.96	1.94	1.89	1.84	1.79	1.77	1.74	1.71	1.68	1.64	1.61
21	2.96	2.57	2.36	2.23	2.14	2.08	2.02	1.98	1.95	1.92	1.87	1.83	1.78	1.75	1.72	1.69	1.66	1.62	1.59
22	2.95	2.56	2.35	2.22	2.13	2.06	2.01	1.97	1.93	1.90	1.86	1.81	1.76	1.73	1.70	1.67	1.64	1.60	1.57
23	2.94	2.55	2.34	2.21	2.11	2.05	1.99	1.95	1.92	1.89	1.84	1.80	1.74	1.72	1.69	1.66	1.62	1.59	1.55
24	2.93	2.54	2.33	2.19	2.10	2.04	1.98	1.94	1.91	1.88	1.83	1.78	1.73	1.70	1.67	1.64	1.61	1.57	1.53
25	2.92	2.53	2.32	2.18	2.09	2.02	1.97	1.93	1.89	1.87	1.82	1.77	1.72	1.69	1.66	1.63	1.59	1.56	1.52
26	2.91	2.52	2.31	2.17	2.08	2.01	1.96	1.92	1.88	1.86	1.81	1.76	1.71	1.68	1.65	1.61	1.58	1.54	1.50
27	2.90	2.51	2.30	2.17	2.07	2.00	1.95	1.91	1.87	1.85	1.80	1.75	1.70	1.67	1.64	1.60	1.57	1.53	1.49
28	2.89	2.50	2.29	2.16	2.06	2.00	1.94	1.90	1.87	1.84	1.79	1.74	1.69	1.66	1.63	1.59	1.56	1.52	1.48
29	2.89	2.50	2.28	2.15	2.06	1.99	1.93	1.89	1.86	1.83	1.78	1.73	1.68	1.65	1.62	1.58	1.55	1.51	1.47
30	2.88	2.49	2.28	2.14	2.03	1.98	1.93	1.88	1.85	1.82	1.77	1.72	1.67	1.64	1.61	1.57	1.54	1.50	1.46
40	2.84	2.44	2.23	2.09	2.00	1.93	1.87	1.83	1.79	1.76	1.71	1.66	1.61	1.57	1.54	1.51	1.47	1.42	1.38
60	2.79	2.39	2.18	2.04	1.95	1.87	1.82	1.77	1.74	1.71	1.66	1.60	1.54	1.51	1.48	1.44	1.40	1.35	1.29
120	2.75	2.35	2.13	1.99	1.90	1.82	1.77	1.72	1.68	1.65	1.60	1.55	1.48	1.45	1.41	1.37	1.32	1.26	1.19
∞	2.71	2.30	2.08	1.94	1.85	1.77	1.72	1.67	1.63	1.60	1.55	1.49	1.42	1.38	1.34	1.30	1.24	1.17	1.00

Degrees of freedom for the denominator (ν_2)

Degrees of freedom for the numerator (ν_1)

Table V Percentage Points of the F Distribution (*continued*)

$F_{0.05,\nu_1,\nu_2}$

ν_2 \ ν_1	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
	Degrees of freedom for the numerator (ν_1)																		
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9	243.9	245.9	248.0	249.1	250.1	251.1	252.2	253.3	254.3
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.43	19.45	19.45	19.46	19.47	19.48	19.49	19.50
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.36
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.07	1.99	1.95	1.90	1.85	1.80	1.75	1.69
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13	2.06	1.97	1.93	1.88	1.84	1.79	1.73	1.67
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.77	1.71	1.65
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.10	2.03	1.94	1.90	1.85	1.81	1.75	1.70	1.64
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39
120	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.61	1.55	1.55	1.43	1.35	1.25
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00

Degrees of freedom for the denominator (ν_2)

(*continues*)

Table V Percentage Points of the F Distribution (*continued*)

$F_{0.025,\,\nu_1,\,\nu_2}$

ν_2 \ ν_1	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	647.8	799.5	864.2	899.6	921.8	937.1	948.2	956.7	963.3	968.6	976.7	984.9	993.1	997.2	1001	1006	1010	1014	1018
2	38.51	39.00	39.17	39.25	39.30	39.33	39.36	39.37	39.39	39.40	39.41	39.43	39.45	39.46	39.46	39.47	39.48	39.49	39.50
3	17.44	16.04	15.44	15.10	14.88	14.73	14.62	14.54	14.47	14.42	14.34	14.25	14.17	14.12	14.08	14.04	13.99	13.95	13.90
4	12.22	10.65	9.98	9.60	9.36	9.20	9.07	8.98	8.90	8.84	8.75	8.66	8.56	8.51	8.46	8.41	8.36	8.31	8.26
5	10.01	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.68	6.62	6.52	6.43	6.33	6.28	6.23	6.18	6.12	6.07	6.02
6	8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.52	5.46	5.37	5.27	5.17	5.12	5.07	5.01	4.96	4.90	4.85
7	8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.82	4.76	4.67	4.57	4.47	4.42	4.36	4.31	4.25	4.20	4.14
8	7.57	6.06	5.42	5.05	4.82	4.65	4.53	4.43	4.36	4.30	4.20	4.10	4.00	3.95	3.89	3.84	3.78	3.73	3.67
9	7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10	4.03	3.96	3.87	3.77	3.67	3.61	3.56	3.51	3.45	3.39	3.33
10	6.94	5.46	4.83	4.47	4.24	4.07	3.95	3.85	3.78	3.72	3.62	3.52	3.42	3.37	3.31	3.26	3.20	3.14	3.08
11	6.72	5.26	4.63	4.28	4.04	3.88	3.76	3.66	3.59	3.53	3.43	3.33	3.23	3.17	3.12	3.06	3.00	2.94	2.88
12	6.55	5.10	4.47	4.12	3.89	3.73	3.61	3.51	3.44	3.37	3.28	3.18	3.07	3.02	2.96	2.91	2.85	2.79	2.72
13	6.41	4.97	4.35	4.00	3.77	3.60	3.48	3.39	3.31	3.25	3.15	3.05	2.95	2.89	2.84	2.78	2.72	2.66	2.60
14	6.30	4.86	4.24	3.89	3.66	3.50	3.38	3.29	3.21	3.15	3.05	2.95	2.84	2.79	2.73	2.67	2.61	2.55	2.49
15	6.20	4.77	4.15	3.80	3.58	3.41	3.29	3.20	3.12	3.06	2.96	2.86	2.76	2.70	2.64	2.59	2.52	2.46	2.40
16	6.12	4.69	4.08	3.73	3.50	3.34	3.22	3.12	3.05	2.99	2.89	2.79	2.68	2.63	2.57	2.51	2.45	2.38	2.32
17	6.04	4.62	4.01	3.66	3.44	3.28	3.16	3.06	2.98	2.92	2.82	2.72	2.62	2.56	2.50	2.44	2.38	2.32	2.25
18	5.98	4.56	3.95	3.61	3.38	3.22	3.10	3.01	2.93	2.87	2.77	2.67	2.56	2.50	2.44	2.38	2.32	2.26	2.19
19	5.92	4.51	3.90	3.56	3.33	3.17	3.05	2.96	2.88	2.82	2.72	2.62	2.51	2.45	2.39	2.33	2.27	2.20	2.13
20	5.87	4.46	3.86	3.51	3.29	3.13	3.01	2.91	2.84	2.77	2.68	2.57	2.46	2.41	2.35	2.29	2.22	2.16	2.09
21	5.83	4.42	3.82	3.48	3.25	3.09	2.97	2.87	2.80	2.73	2.64	2.53	2.42	2.37	2.31	2.25	2.18	2.11	2.04
22	5.79	4.38	3.78	3.44	3.22	3.05	2.93	2.84	2.76	2.70	2.60	2.50	2.39	2.33	2.27	2.21	2.14	2.08	2.00
23	5.75	4.35	3.75	3.41	3.18	3.02	2.90	2.81	2.73	2.67	2.57	2.47	2.36	2.30	2.24	2.18	2.11	2.04	1.97
24	5.72	4.32	3.72	3.38	3.15	2.99	2.87	2.78	2.70	2.64	2.54	2.44	2.33	2.27	2.21	2.15	2.08	2.01	1.94
25	5.69	4.29	3.69	3.35	3.13	2.97	2.85	2.75	2.68	2.61	2.51	2.41	2.30	2.24	2.18	2.12	2.05	1.98	1.91
26	5.66	4.27	3.67	3.33	3.10	2.94	2.82	2.73	2.65	2.59	2.49	2.39	2.28	2.22	2.16	2.09	2.03	1.95	1.88
27	5.63	4.24	3.65	3.31	3.08	2.92	2.80	2.71	2.63	2.57	2.47	2.36	2.25	2.19	2.13	2.07	2.00	1.93	1.85
28	5.61	4.22	3.63	3.29	3.06	2.90	2.78	2.69	2.61	2.55	2.45	2.34	2.23	2.17	2.11	2.05	1.98	1.91	1.83
29	5.59	4.20	3.61	3.27	3.04	2.88	2.76	2.67	2.59	2.53	2.43	2.32	2.21	2.15	2.09	2.03	1.96	1.89	1.81
30	5.57	4.18	3.59	3.25	3.03	2.87	2.75	2.65	2.57	2.51	2.41	2.31	2.20	2.14	2.07	2.01	1.94	1.87	1.79
40	5.42	4.05	3.46	3.13	2.90	2.74	2.62	2.53	2.45	2.39	2.29	2.18	2.07	2.01	1.94	1.88	1.80	1.72	1.64
60	5.29	3.93	3.34	3.01	2.79	2.63	2.51	2.41	2.33	2.27	2.17	2.06	1.94	1.88	1.82	1.74	1.67	1.58	1.48
120	5.15	3.80	3.23	2.89	2.67	2.52	2.39	2.30	2.22	2.16	2.05	1.94	1.82	1.76	1.69	1.61	1.53	1.43	1.31
∞	5.02	3.69	3.12	2.79	2.57	2.41	2.29	2.19	2.11	2.05	1.94	1.83	1.71	1.64	1.57	1.48	1.39	1.27	1.00

Degrees of freedom for the numerator (ν_1)

Degrees of freedom for the denominator (ν_2)

Table V Percentage Points of the F Distribution (*continued*)

$F_{0.01,\nu_1,\nu_2}$

ν_2 \ ν_1	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	4052	4999.5	5403	5625	5764	5859	5928	5982	6022	6056	6106	6157	6209	6235	6261	6287	6313	6339	6366
2	98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.37	99.39	99.40	99.42	99.43	99.45	99.46	99.47	99.47	99.48	99.49	99.50
3	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.35	27.23	27.05	26.87	26.69	26.00	26.50	26.41	26.32	26.22	26.13
4	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66	14.55	14.37	14.20	14.02	13.93	13.84	13.75	13.65	13.56	13.46
5	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16	10.05	9.89	9.72	9.55	9.47	9.38	9.29	9.20	9.11	9.02
6	13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87	7.72	7.56	7.40	7.31	7.23	7.14	7.06	6.97	6.88
7	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62	6.47	6.31	6.16	6.07	5.99	5.91	5.82	5.74	5.65
8	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81	5.67	5.52	5.36	5.28	5.20	5.12	5.03	4.95	4.86
9	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26	5.11	4.96	4.81	4.73	4.65	4.57	4.48	4.40	4.31
10	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85	4.71	4.56	4.41	4.33	4.25	4.17	4.08	4.00	3.91
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54	4.40	4.25	4.10	4.02	3.94	3.86	3.78	3.69	3.60
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30	4.16	4.01	3.86	3.78	3.70	3.62	3.54	3.45	3.36
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10	3.96	3.82	3.66	3.59	3.51	3.43	3.34	3.25	3.17
14	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03	3.94	3.80	3.66	3.51	3.43	3.35	3.27	3.18	3.09	3.00
15	8.68	6.36	5.42	4.89	4.36	4.32	4.14	4.00	3.89	3.80	3.67	3.52	3.37	3.29	3.21	3.13	3.05	2.96	2.87
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69	3.55	3.41	3.26	3.18	3.10	3.02	2.93	2.84	2.75
17	8.40	6.11	5.18	4.67	4.34	4.10	3.93	3.79	3.68	3.59	3.46	3.31	3.16	3.08	3.00	2.92	2.83	2.75	2.65
18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	3.51	3.37	3.23	3.08	3.00	2.92	2.84	2.75	2.66	2.57
19	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43	3.30	3.15	3.00	2.92	2.84	2.76	2.67	2.58	2.59
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37	3.23	3.09	2.94	2.86	2.78	2.69	2.61	2.52	2.42
21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40	3.31	3.17	3.03	2.88	2.80	2.72	2.64	2.55	2.46	2.36
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26	3.12	2.98	2.83	2.75	2.67	2.58	2.50	2.40	2.31
23	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30	3.21	3.07	2.93	2.78	2.70	2.62	2.54	2.45	2.35	2.26
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17	3.03	2.89	2.74	2.66	2.58	2.49	2.40	2.31	2.21
25	7.77	5.57	4.68	4.18	3.85	3.63	3.46	3.32	3.22	3.13	2.99	2.85	2.70	2.62	2.54	2.45	2.36	2.27	2.17
26	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18	3.09	2.96	2.81	2.66	2.58	2.50	2.42	2.33	2.23	2.13
27	7.68	5.49	4.60	4.11	3.78	3.56	3.39	3.26	3.15	3.06	2.93	2.78	2.63	2.55	2.47	2.38	2.29	2.20	2.10
28	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12	3.03	2.90	2.75	2.60	2.52	2.44	2.35	2.26	2.17	2.06
29	7.60	5.42	4.54	4.04	3.73	3.50	3.33	3.20	3.09	3.00	2.87	2.73	2.57	2.49	2.41	2.33	2.23	2.14	2.03
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98	2.84	2.70	2.55	2.47	2.39	2.30	2.21	2.11	2.01
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80	2.66	2.52	2.37	2.29	2.20	2.11	2.02	1.92	1.80
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63	2.50	2.35	2.20	2.12	2.03	1.94	1.84	1.73	1.60
120	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	2.47	2.34	2.19	2.03	1.95	1.86	1.76	1.66	1.53	1.38
∞	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41	2.32	2.18	2.04	1.88	1.79	1.70	1.59	1.47	1.32	1.00

Degrees of freedom for the denominator (ν_2)

Degrees of freedom for the numerator (ν_1)

Table XIII Factors for Quality-Control Charts

	$\bar{X}$ Chart		R Chart		
	Factors for Control Limits		Factors for Central Line	Factors for Control Limits	
n[a]	A_1	A_2	d_2	D_3	D_4
2	3.760	1.880	1.128	0	3.267
3	2.394	1.023	1.693	0	2.575
4	1.880	0.729	2.059	0	2.282
5	1.596	0.577	2.326	0	2.115
6	1.410	0.483	2.534	0	2.004
7	1.277	0.419	2.704	0.076	1.924
8	1.175	0.373	2.847	0.136	1.864
9	1.094	0.337	2.970	0.184	1.816
10	1.028	0.308	3.078	0.223	1.777
11	0.973	0.285	3.173	0.256	1.744
12	0.925	0.266	3.258	0.284	1.716
13	0.884	0.249	3.336	0.308	1.692
14	0.848	0.235	3.407	0.329	1.671
15	0.816	0.223	3.472	0.348	1.652
16	0.788	0.212	3.532	0.364	1.636
17	0.762	0.203	3.588	0.379	1.621
18	0.738	0.194	3.640	0.392	1.608
19	0.717	0.187	3.689	0.404	1.596
20	0.697	0.180	3.735	0.414	1.586
21	0.679	0.173	3.778	0.425	1.575
22	0.662	0.167	3.819	0.434	1.566
23	0.647	0.162	3.858	0.443	1.557
24	0.632	0.157	3.895	0.452	1.548
25	0.619	0.153	3.931	0.459	1.541

[a] $n > 25$; $A_1 = 3/\sqrt{n}$. n = number of observations in sample.