

KATHMANDU UNIVERSITY
End Semester Examination [C]
April/May 2023

Marks Scored:

Level : B.E./B.Sc.

Year : II

Exam Roll No. :

Time: 30 mins.

Course : MATH 208

Semester : I

F. M. : 20

Date : 27 APR 2023

Registration No.:

SECTION "A"

[10Q. \times 1 = 10 marks]

Fill in the blank(s) by the most appropriate word(s) or symbol(s).

1. The square of standard deviation is called _____.
2. The minimum value, first quartile, median, third quartile and maximum value are used to construct _____ plot.
3. A bag contains 9 red and 6 white balls, if two balls are drawn one after another without replacement, then the probability of getting a red ball in second draw given that the first ball drawn was also red is _____.
4. If $V(X) = 2.5$, then $V(2X) =$ _____.
5. If $X \sim B(12, 0.4)$, then $P(X \geq 1) =$ _____.
6. If $X \sim N(15, 3^2)$, then $P(\text{_____} \leq X \leq \text{_____}) = 0.6826$.
7. To test equality of means of two normal populations, when population variances are not known and sample sizes are less than 30, _____ distribution is used.
8. The error that is committed in accepting the null hypothesis when, in fact, it is wrong is called _____ error.
9. The expression for lower control line to construct \bar{X} -chart based on sample range is $LCL =$ _____.
10. In simple linear regression, the relationship between dependent (y) and independent variable (x) is modeled by equation _____.

SECTION "B"

[10Q. \times 1 = 10 marks]

Fill in the blank space(s), **DO NOT TICK**, by selecting the most appropriate answers from among the given ones.

11. For a symmetric distribution _____ lies between the 1st and the 3rd quartile.
(i) mean (ii) median (iii) mode (iv) standard deviation

12. If $P(A \cup B) = P(A) + P(B)$, then events A and B are called _____.
- (i) independent (ii) mutually exclusive
(iii) dependent (iv) conditional
13. If X is a continuous random variable with probability density function $f(x)$, then $E(X^2) =$ _____.
- (i) $\int x \cdot f(x^2) dx$ (ii) $\int x^2 \cdot f(x) dx$ (iii) $\int x^2 \cdot f(x^2) dx$ (iv) $(\int x \cdot f(x))^2 dx$
14. It is known that 3 requests enter into a server on average in 1 second. The probability distribution of number of requests entering into the server can be modeled by _____ distribution.
- (i) binomial (ii) Poisson (iii) normal (iv) chi-square
15. If $X \sim N(\mu, \sigma^2)$ and $Y = 2X + 3$, then the distribution of Y is _____.
- (i) $N(2\mu, 2\sigma^2)$ (ii) $N(2\mu, 4\sigma^2)$ (iii) $N(2\mu + 3, 4\sigma^2)$ (iv) $N(2\mu, 4\sigma^2 + 3)$
16. If a sample of n units is selected randomly from a $N(\mu, \sigma^2)$ population, then $\frac{\sigma^2}{n}$ represents _____.
- (i) variance of sample observations (ii) variance of sample means
(iii) standard deviation of sample means (iv) skewness of sample observations
17. If \bar{X} and S^2 are the mean and variance of a sample of size n drawn from $N(\mu, \sigma^2)$ population, then the statistic _____ has χ_{n-1}^2 distribution.
- (i) $\frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$ (ii) $\frac{\bar{X} - \mu}{S/\sqrt{n}}$ (iii) $\frac{(n-1)S^2}{\sigma^2}$ (iv) $\frac{S^2}{\sigma^2}$
18. A researcher wants to test whether the resistance of electric wire can be reduced by more than 0.050 Ohm by alloying. In fact, the test is _____ tailed.
- (i) left (ii) right (iii) two (iv) heavy
19. The error caused by _____ is a factor of random cause of variation.
- (i) faulty equipment (ii) defective raw material
(iii) un-experienced operator (iv) minor mistakes in operation
20. In simple linear regression equation $y = 21.5 + 1.24x$, the predicted value of dependent variable for value 4.8 of independent variable is _____.
- (i) 21.5 (ii) 1.24 (iii) 5.592 (iv) 27.452

KATHMANDU UNIVERSITY
End Semester Examination [C]
April/May 2023

27 APR 2023

Level : B.E./B.Sc.
Year : II
Time : 2 hrs. 30 mins.

Course : MATH 208
Semester : I
F. M. : 55

SECTION "C"
[3Q. × 7 = 21 marks]

1. Direct evidence of Newton's universal law of gravitation was provided from a renowned experiment by Henry Cavendish. In the experiment, masses of objects were determined by weighing, and the measured force of attraction was used to calculate the density of the earth. The values of the earth's density, in time order by row, are

5.36	5.29	5.58	5.65	5.57	5.53	5.62	5.29
5.44	5.34	5.79	5.10	5.27	5.39	5.42	5.47
5.63	5.34	5.46	5.30	5.75	5.68	5.85	5.32

- a. Construct stem-and-leaf plot of above data. [1]
 - b. Use the result of part (a) to construct ordered stem-and-leaf plot and list the data in ascending order. [1]
 - c. Determine median and quartiles using result in part (b). [3]
 - d. Construct boxplot and interpret the nature of given data. [2]
2. 10 cars and 10 pickup trucks were selected randomly to observe the number of years that they were kept by original owner before being sold and following results were obtained.

Cars	5	1	4	7	1	9	5	3	5	4
Trucks	6	5	5	10	7	7	11	7	6	6

Assuming that the number of years that the original owners of both cars and trucks before being sold are normally distributed with more or less same variance, conduct an investigation to test whether there is any difference in the mean time in years of cars and of trucks before they are sold by the owners at 5% level of significance. Also construct the 95% confidence interval for the difference in the means based on these data.

OR

10 cars were equipped with radial tires and driven over a test course. Then the same 10 cars (with the same drivers) were equipped with regular belted tires and driven over the same course. After each run, the cars' fuel economy (in km/l) was measured. The results are presented below-

Radial tire	4.2	4.7	6.6	7.0	6.8	4.5	5.7	6.0	7.4	4.9
Belted tire	4.1	4.9	6.2	6.9	6.8	4.4	5.7	5.8	6.9	4.7

Is there evidence that radial tires produce better fuel economy? (Assume normality of data, and use paired t-test at $\alpha = 0.05$.) Also, find a 95% confidence interval for the mean difference in fuel economy between radial tires and belted tires.

3. A company manufactures an electronic device to be used in a very wide temperature range. The company knows that increased temperature shortens the life time of the device, and a study is therefore performed in which the life time is determined as a function of temperature. The following data is found:

Temperature in Celsius (x)	10	20	30	40	50	60	70	80	90
Life time in hours (y)	420	365	285	220	176	117	69	34	5

- a. Construct a scatter plot to show that it is appropriate to apply linear regression. [1]
 b. Develop the linear regression model to establish relationship between temperature and life-time. [3]
 c. Predict the life time for the temperature of $0^{\circ}C$. [1]
 d. Find correlation coefficient between temperature and life-time and interpret it. [2]

SECTION "D"

[6Q. \times 4 = 24 marks]

4. Three identical boxes contain red and white balls. The first box contains 3 red and 2 white balls, the second box has 4 red and 5 white balls, and the third box has 2 red and 4 white balls. A box is chosen very randomly and a ball is drawn from it. If the ball that is drawn out is red, what will be the probability that the second box is chosen?
5. X, the random variable denoting the number of breads that certain bakery is able to sell on a day had probability density function
- $$f(x) = \begin{cases} Ax; & 0 \leq x \leq 500 \\ A(10 - x); & 500 \leq x \leq 1000 \\ 0; & \text{otherwise} \end{cases}$$
- Find the constant A and the probability that the number of breads that will be sold tomorrow will be between 250 and 750.
6. The number of customers arriving at a grocery store is a Poisson random variable. On average 10 customers arrive per hour. Let X be the number of customers arriving from 10 am to 11:30 am. Find $P(12 < X \leq 15)$.
7. The time, X, in minutes taken to install a satellite dish is assumed to be a normal random variable with mean 134 and standard deviation 16. (a) Determine $P(X < 150)$. (b) Determine, to one decimal place, the time exceeded by 10% of installations.
8. It is reported that the population standard deviation of scores on the quantitative portion of the Graduate Record Examination (GRE) General Test for students taking the exam between 1994 and 1997 was 139. A sample of 100 participants is observed and sample mean was observed to be 585. Carry the one-independent sample z-test for whether or not we will retain the null hypothesis that population mean of GRE-scores is more than 560 at a 0.05 level of significance. Also, construct 95% two-sided confidence interval for the population mean based on sample observations.

9. The thickness of silicon chips produced by a machine are observed. 12 samples each of size 3 are found to have following values of means and ranges of thickness of sample units:

Sample	1	2	3	4	5	6	7	8	9	10	11	12
Means (\bar{X})	0.258	0.249	0.259	0.268	0.256	0.268	0.260	0.257	0.257	0.245	2.248	0.239
Range (R)	0.038	0.030	0.009	0.024	0.039	0.003	0.026	0.046	0.035	0.017	0.049	0.005

Construct (a) \bar{X} based on range (b) R chart. Analyze the charts. [Hints: $A_2 = 1.023$, $D_3 = 0$, $D_4 = 2.574$]

SECTION "E"

[5Q. \times 2 = 10 marks]

10. In tossing of an unbiased coin 6 times, find probability of getting at the least 5 heads.
11. Two hundred patients who had either hip surgery or knee surgery were asked whether they were satisfied or dissatisfied regarding the result of their surgery. The following table summarizes their response-

Surgery	Satisfied	Dissatisfied
Knee	70	25
Hip	90	15

If one person from the 200 patients is selected at random, determine the probability that the person was satisfied GIVEN that the person had knee surgery.

12. Mention the expressions for test statistics that are used to test hypothesis on standard deviation of (a) one population (b) two populations. Describe the meaning of each symbol used in the expressions and also state what distributions these statistics follow.
13. A researcher is interested in studying average weight of obese children in a province. Previous literature gives an estimated standard deviation of weight of obese children in the province to be 1.24. What is the sample size required for him/her to observe if 95% confidence level is required and that maximum tolerable error is 1.5?
14. Write short notes on:
- assignable causes of variation
 - p-chart

