Marks scored:

KATHMANDU UNIVERSITY
End Semester Examination [C]
January, 2019

Level : B.E./B. Sc.	Course : MATH 207
Year : II	Semester: II
Exam Roll No. : Time: 30 mins	F. M. : 20
Registration No:	Date : JAN 04 2019

SECTION "A"
[10 Q. × 1=10 marks]

Fill in the blank space(s) by writing the most appropriate word(s) or symbol(s).

1. The differential equation $(2x + 3y)dx + (ax + by)dy$ will be exact if $a =$......................., where a, b are constants.

2. If $y = \sin x$ is a solution of the differential equation $\frac{d^2y}{dx^2} + y = c$, then $c =$ ....

3. The Wronskian of the solutions $y_1 = e^x$ and $y_2 = e^{-x}$ of the second order linear differential equation $\frac{d^2y}{dx^2} - y = 0$ is ....

4. The Laplace transform of the function $f(t) = \cos 3t$ is ....

5. The convolution $(1 * 1) =$ ....

6. The partial differential equation (PDE) $a\frac{\partial^2 u}{\partial x^2} + b\frac{\partial^2 u}{\partial x \partial y} + c\frac{\partial^2 u}{\partial y^2} = 0$ will be parabolic type in PDE classification if ....

7. The principal value of the argument of the complex number $z = 2 + 2i$, where $i = \sqrt{-1}$ denotes the imaginary unit, is $Arg z =$........................ .

8. The imaginary part of the complex valued function $f(z) = z^2$ at $z = 2 + 3i$ is ....

9. The complex integration $\int_C z\, dz =$................., where C is the straight line segment from $z = 0$ to i .

10. If the function $f(z)$ has Laurent series expansion $\frac{1}{z^3} - \frac{1}{3!z} + \frac{z}{5!} - \frac{z^3}{7!} + \cdots$, then the residue of $f(z)$ at the pole $z = 0$ is ....

SECTION "B"
[10 Q.×1 = 10 marks]

Fill in the blank space(s), **DO NOT TICK**, by selecting the most appropriate answers from among the given ones.

11. The Laplace equation $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$, where $f = f(x, y)$, is the.................order partial differential equation.
[zeroth; first; second; third]

12. The Bernoulli equation $\frac{dy}{dx} + p(x)y = r(x)y^{\alpha}, \alpha \neq 0$ and $\alpha \neq 1$, reduces to the linear differential equation in a new variable $u = u(x)$ with the substitution ....

[$u = y^{1-\alpha}$; $u = y^{-\alpha}$; $u = y^{\alpha}$; $u = y^{1+\alpha}$]

13. The number of basis solutions of the second order differential equation $\frac{d^2y}{dx^2} + p(x)\frac{dy}{dx} + q(x)y = r(x)$ is ....

[0; 1; 2; 3]

14. For an integer n, the Bessel's functions $J_{-n}(x)$ and $J_n(x)$ are ....

[both zeros; linearly dependent; linearly independent; not defined]

15. The inverse Laplace transform $\mathcal{L}^{-1}(e^{-as}) =$ ....

[1; $U(t-a)$; $\delta(t-a)$; 0]

16. The one-dimensional wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ is .... type in PDE classification.

[circular; elliptic; parabolic; hyperbolic]

17. If $i = \sqrt{-1}$ denotes the imaginary unit in the complex plane, then $i^9 =$.................. .

[i; $-i$; 1; -1]

18. The region in the complex plane defined by $\{z \in \mathbb{C} \mid 2 < |z| < 3\}$ is the .... domain.

[simply connected; doubly connected; triply connected; not connected]

19. If the power series $\sum_{n=0}^{\infty} \frac{(2n)!}{(n!)^2}(z-3i)^n$ has radius of convergence $R = 0.25$, then the region of convergence of the series is ....

[$|z-3i| = 0.25$; $|z-3i| > 0.25$; $|z-3i| < 0.25$; empty set]

20. The function $f(z) = z(z-1)^3(z-i)^2$ has a zero of order .... at $z = i$.

[0; 1; 2; 3]

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JAN 0 4 2019

Level : B.E./B. Sc.
Year : II
Time : 2 hrs. 30 mins.

Course : MATH 207
Semester: II
F. M. : 55

SECTION "C"
[3 Q. × 7 = 21 marks]

1. Define the second order linear ordinary differential equation (ODE). Explain the procedure to find the general solution of linear second order homogeneous ODE with constant coefficients and discuss the different possible solutions of this equation. Solve the initial value Euler-Cauchy equation $x^2\frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + 6y = 0, y(1) = 1, y'(1) = 0$. [1+3+3]

2. Define the convolution of two functions. State and prove the convolution theorem for Laplace transform. Find the inverse Laplace transform of $H(s) = \frac{1}{s^2(s^2+1)}$ using convolution theorem. [1+4+2]

OR

Discuss three different types of second order partial differential equations (PDE) of the form $A\frac{\partial^2 u}{\partial x^2} + 2B\frac{\partial^2 u}{\partial x \partial y} + C\frac{\partial^2 u}{\partial y^2} = F(x, y, u, \frac{\partial u}{\partial x}, \frac{\partial u}{\partial y})$ conditioned on a relation of the coefficients A, B, C. Classify the type of the partial differential equation $\frac{\partial^2 u}{\partial x^2} - 4\frac{\partial^2 u}{\partial x \partial y} + 4\frac{\partial^2 u}{\partial y^2} = 0$ and transform this equation into normal form and solve it. (Show the detail work) [2+5]

3. Derive the Cauchy-Riemann equations for an analytic function $f(z)$ in a domain D. Verify that $u = x^2 - y^2 - y$ is a harmonic function and then find a harmonic conjugate function of u. [3+4]

SECTION "D"
[6 Q. × 4= 24 marks]

4. Find the solution of the first order linear differential equation $x^2\frac{dy}{dx} + 3xy = \frac{1}{x}$ with initial condition $y(1) = -1$.

5. Show that $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\sin x$, where $J_\nu(x) = \sum_{m=0}^{\infty} \frac{(-1)^m x^{2m+\nu}}{2^{2m+\nu} m!\,\Gamma(\nu+m+1)}$ is the Bessel's function of first kind of order $\nu = \frac{1}{2}$.

6. Express the function $f(t) = \begin{cases} t^2, & 1 < t < 2 \\ 0, & otherwise \end{cases}$ using unit step functions and find its Laplace transform.

7. Integrate $\oint_C \bar{z}\, dz$ in the path C from $z_0 = -1 + i$ along the parabola $y = x^2$ to $z_1 = 1 + i$.

8. Find the residues of the function $f(z) = \frac{30z^2-23z+5}{(2z-1)^2(3z-1)}$ at the singularities. Use these residue(s) to evaluate the complex integration $\oint_C \frac{30z^2-23z+5}{(2z-1)^2(3z-1)} dz$ along the unit circle $C: |z| = 1$.

OR

Evaluate the real integration $\int_0^{2\pi} \frac{d\theta}{5-4\sin\theta}$ by applying residue theorem.

9. Consider an *RLC-circuit* with negligibly small resistance R. Find the current in the circuit composed of an $L = 0.2\ H$ inductor, $C = 0.05\ F$ capacitor and electromotive force $E = 1\ Volt$, assuming zero initial current and zero initial charge.

SECTION "E"

[5 Q. × 2= 10 marks]

10. Find the power series solution of the first order differential equation $\frac{dy}{dx} = -y$.

11. Find the orthogonal trajectories of the family of lines $y = Cx$, where C is an arbitrary constant.

12. Find the square roots of the complex number $z = -i$, where $i = \sqrt{-1}$ denotes the imaginary unit.

13. Find the linear fractional transformation that maps the points $0, 1, \infty$ onto $\infty, 1, 0$.

14. Develop the Taylor's series of the function $f(z) = \frac{1}{z}$ centered at $z_0 = i$.