Marks Scored

KATHMANDU UNIVERSITY
End Semester Examination
May/June, 2022

Level : B.E./B.Sc.
Year : II

Course : MATH 207
Semester : II

Exam Roll No.:

Time: 30 mins

F.M. : 20

Registration No:

Date :

SECTION "A"

[10Q. × 1 = 10 marks]

Fill in the blank space(s) by writing most appropriate word(s) or symbol(s)

1. The integrating factor of the differential equation $y' + py = r$ is ____________.

2. Two Linearly independent solutions of the ODE $y'' - 9\pi^2 y = 0$ are____________ and ____________.

3. Suppose the two functions y_1 and y_2 are linearly dependent, then $W(y_1, y_2) =$ ____________.

4. The radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{n!}{n^n}(z+\pi)^n$ is ____________.

5. The Laplace transform of $e^t \cos t$ is ____________.

6. $(f * g)(t) =$ ____________.

7. The Legendre's polynomial $P_3(x)$ is ____________.

8. In the neighborhood of $z = 1$, the function $f(z)$ has a power series expansion of the form $f(z) = 1 + (1-z) + (1-z)^2 + \cdots \infty$. Then $f(z) =$ ____________.

9. If $u = x^2 - y^2$, then the conjugate harmonic function is ____________.

10. If $f(z)$ is analytic within and on a closed curve C and if z_0 is any point within C, then $\int_C \frac{f(z)}{z-z_0} dz =$____________.

SECTION "B"

[10Q. × 1 = 10 marks]

Fill in the blank space(s), **DO NOT TICK**, by selecting the most appropriate answers from among the given ones.

11. If $y = x^2$ is a solution of the differential equation $x^2 y'' + y = f(x)$, then $f(x) =$ ____________.

    [x^2; $3x^2$; $-x^2$; $x^2 + 3x$]

12. The differential equation $2y\,dx - (3y - 2x)dy = 0$ is ___________.
[exact, homogenous and linear; homogenous and linear but not exact;
Exact and linear but not homogenous; exact and homogenous but not linear]

13. For integer $\nu = n$, $J_{-n}(x) =$ ___________, where $J_n(x)$ is the Bessel's polynomial of order n.
[$(-1)^n J_n(x)$; $(-1)^n J_{-n}(x)$; $J_n(x) + J_{-n}(x)$; $J_{-n}(x)$]

14. The Laplace transform of dirac delta function $\delta(t-1)$ is ___________.
[$\frac{e^s}{s}$; $\frac{e^{-s}}{a}$; e^s; e^{-s}]

15. The solution $u(x,y)$ of partial differential equation $u_{xy} = -u_x$ is ___________.
[$f(x)e^{-y} + g(y)$; $f(x)e^{y} + g(x)$; $f(y)e^{x} + g(y)$; $f(y)e^{-x} + g(x)$]

16. The order and degree of the differential equation whose solution is given by $c_1 \cos t + c_2 \sin t$ is___________.
[Order = 2, degree = 1; Order = 1, degree = 1;
Order = 2, degree = 2; Order = 1, degree = 1]

17. The wave equation $u_{tt} - u_{xx} = 0$ is ___________.
[elliptic; parabolic; hyperbolic; circular]

18. The argument of the complex number i is ___________.
[0; $\frac{\pi}{2}$; $-\pi$; 1]

19. The residue of $f(z) = \frac{1+2z}{z^2-z-2}$ at $z = 2$ is___________.
[$-\frac{5}{3}$; $-\frac{1}{3}$; $\frac{2}{3}$; $\frac{5}{3}$]

20. For the analytic function $z \sin\left(\frac{1}{z}\right)$; $z = 0$ is a ___________.
[Pole of order 2; Isolated Singularity;
Essential Singularity; Removable Singularity]

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Level : B.E./B.Sc.
Year : II
Time : 2 hrs. 30 mins

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F.M. : 55

SECTION "C"
[3Q. × 7 = 21 marks]

1. Explain the variation of parameter method to find the solution of
$$y'' + p(x)y' + q(x)y = r(x).$$
Using this method, solve $y'' - 2y' + y = \frac{e^x}{x^3}$. [3+4]

2. State the convolution theorem. Find $1 * \sin t$. Using the Laplace transform, solve the initial value problem $y'' + 4y' + 5y = \delta(t-1),\ y(0) = 0, y'(0) = 3$. [1+1+5]

3. If a function $f(z)$ be analytic in a simply connected domain D and z_1 and z_2 be any two complex numbers in D then show that $\int_{z_1}^{z_2} f(z)dz$ is independent of path joining these two numbers. Evaluate the integral $\int_C \ln z\ dz,\ C: |z| = 1$ (Counterclockwise) [3+4]

OR

State and prove Cauchy Residue theorem for complex integration. Evaluate $\int_C \frac{4-3z}{z(z-1)(z-2)} dz$ where C is the circle $|z| = 1.5$. [3+4]

SECTION "D"
[6Q. × 4 = 24 marks]

4. Show that $J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\ \cos x$.

5. Show that the function $u(x,y) = 3x^2y - y^3$ is harmonic function and find the harmonic conjugate $v(x,y)$. Also, find $f(z) = u(x,y) + iv(x,y)$.

6. Solve the initial value problem $y''' - y'' - y' + y = 0$,
$y(0) = 2,\ y'(0) = 1, y''(0) = 0$.

7. Transform the following equation to normal form and solve $4u_{xx} - u_{yy} = 0$.

OR

Find the solution of the equation $u_x + u_y = (x+y)u$ by separating variables.

8. Express the Laplace equation $u_{xx} + u_{yy} = 0$ into polar form.

9. Evaluate $\int_0^{2\pi} \frac{d\theta}{5 - 3\sin\theta}$ using contour integral.

SECTION "E"

[5Q. × 2 = 10 marks]

10. Find the orthogonal trajectories to the curve $y = ce^{-x}$.

11. Find the inverse transform of the function $\ln\left(\frac{s+a}{s+b}\right)$.

12. Find the linear fractional transformation that maps the points $i, -i, 0$ onto $0, \infty, -1$.

13. Find the general power of $(1+i)^i$.

14. Test for exactness and solve $2xy\,dx + x^2dy = 0$.