KATHMANDU UNIVERSITY
End Semester Examination [C]
May/June, 2019

Marks Scored:

Level: B. E./B. Tech.
Year : II

Course : MATH 207
Semester : I

Exam Roll No. :

Time: 30 mins.

F. M. : 20

Registration No.: .

Date 03 JUN 2019

SECTION "A"

[10Q. × 1 =10 marks]

Fill in the blank space(s) by the most appropriate word(s) or symbol(s).

1. A first order differential equation of the form $y' + p(x)y = r(x)\, y^n, n \in \mathbb{R}$ represents .... equation.

2. The general solution of Cauchy – Euler equation $x^2y'' + axy' + by = 0$ having complex characteristics solutions is ....

3. If $J_\nu(x)$ be the solution of Bessel's equation then $[x^\nu J_\nu(x)]' =$..........................

4. Inverse Laplace transform of $e^{-as}F(s)$, that is $L^{-1}(e^{-as}F(s)) =$.......................

5. If $L(f(t))$ represents Laplace transform of $f(t)$, then $L[t^n f(t)] =$......................

6. The Laplace equation $u_{xx} + u_{yy} = 0$ in polar coordinate system is ....

7. If $z = x + iy$ is a complex number then $\sin iz =$...........................

8. A mapping $w = f(z)$ is conformal if it preserves angles between two oriented curves both in ....

9. The convolution of $e^{kt} * e^{kt}$ is ....

10. The Cauchy-Hadamard formula for finding radius of convergence of a power series $\sum_{n=0}^{\infty} a_n(z - z_0)^n$ is given by the relation ....

SECTION "B"

[10 Q. × 1 = 10 marks]

Fill in the blank space(s), **DO NOT TICK**, by choosing the most appropriate answer from among the given ones.

11. If $F = e^{\int R(y)dy}$ is an integrating factor of a non exact equation $P(x,y)dx + Q(x,y)dy = 0$ then $R(y) =$ ....

    [$\frac{1}{Q}\left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right)$; $\frac{1}{P}\left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)$; $\frac{1}{P}\left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right)$; $\frac{1}{Q}\left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)$]

12. In method of undetermined coefficients for solving $y'' + p(x)y' + q(x)y = r(x)$, if $r(x) = k\cos\omega x$ then the particular solution $y_p(x) =$ .... where K and M are arbitrary constant.

    [$K\cos\omega x$; $K\sin\omega x$; $K\cos\omega x \sin\omega x$; $K\cos\omega x$ +M $\sin\omega x$]

13. Legendre polynomial of degree 0, $P_0(x) =$..................................

[1; x; $\frac{1}{2}(3x^2 - 1)$; $\frac{1}{2}(5x^3 - 3x)$]

14. The Laplace transform of the function $\cosh \omega t$ is ....

[$\frac{s}{s^2+\omega^2}$; $\frac{s}{s^2-\omega^2}$; $\frac{1}{s^2+\omega^2}$; $\frac{1}{s^2-\omega^2}$]

15. If $u = e^{-4t}\cos 8x$ be solution of heat equation $u_t = c^2 u_{xx}$ then $c =$....................

[$\pm\frac{1}{32}$; $\pm\frac{1}{16}$; $\pm\frac{1}{8}$; $\pm\frac{1}{4}$]

16. $e^{\pi i} =$ ....

[1; -1; i; $-i$]

17. $Ln\, i =$......................................

[π; $\frac{\pi}{2}$; πi; $\frac{\pi}{2} i$]

18. The Cauchy principal value of $\int_{-\infty}^{\infty} \frac{dx}{x^2-1}$ is ....

[0; $-\pi i$; $\frac{\pi}{2} i$; πi]

19. The function $f(z) = \frac{1}{(z-1)(z+1)^2(z-i)^3(z+i)^4}$ has pole of order .... at $z = -i$.

[1; 2 3; 4]

20. The fixed point of a mapping $w = (4 + i)z$ is ....

[0; $-i$ -4; $-4 - i$]

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Time : 2 hrs. 30 mins.

Course : MATH 207
Semester : I
F.M. : 55

SECTION "C"
[3 Q. × 7 = 21 marks]

1. Show that $y_p(x) = -y_1 \int \frac{y_2 r}{w} dx + y_2 \int \frac{y_1 r}{w} dx$ be the particular solution of non-homogeneous differential equation $y'' + p(x)y' + q(x)y = r(x)$, where y_1 and y_2 are tow linearly independent solutions of the associated homogeneous equation, and w is Wronskian y_1 and y_2. Use this method to solve the equation $y'' + y = \sec x$. [3 + 4]

2. State and prove first shifting theorem of Laplace transform. Solve using Laplace transform: $y'' + 9y = 10e^{-t}$, $y(0) = 0$, $y'(0) = 0$. [1 + 2 + 4]

OR

State and prove second shifting theorem of Laplace transform. Solve using Laplace transform: $y'' - 4y' + 4y = 0$, $y(0) = 2.1$, $y'(0) = 3.9$ [1 + 2 + 4]

3. Prove that if $f(z) = u(x, y) + iv(x, y)$ is analytic in a domain D then the first order partial derivatives of u and v satisfy Cauchy – Riemann equations $u_x = v_y$, $u_y = -v_x$. Verify that $u = e^{-x} \sin y$ is harmonic and find corresponding analytic function. [3 + 4]

SECTION "D"
[6 Q. × 4 = 24 marks]

4. Solve the initial value problem: $y' + \frac{y}{x^2} = 2xe^{1/x}, y(1) = 13.86$.

5. Prove that $[x^{-\nu} J_\nu(x)]' = -x^{-\nu} J_{\nu+1}(x)$, where $J_\nu(x)$ be the Bessel function of first kind of order γ.

6. Solve using method of undetermined coefficients: $y'' + 3y' + 2.25y = -10\, e^{-1.5x}$.

7. Classify and transform $u_{xx} - 10u_{xy} + 25u_{yy} = 0$ into normal form and hence solve it.

OR

Solve the Laplace equation: $u_{xx} + u_{yy} = 0$.

8. Integrate: $\int_c \bar{z}\, dz$, C from $-1 + i$ along the parabola $y = x^2$ to $1 + i$.

9. Integrate counterclockwise around C: $\oint_C \frac{z^2 \sin z}{4z^2 - 1}$, $C: |z - 1| = 2$.

SECTION "E"
[5Q. × 2 = 10 marks]

10. Find orthogonal trajectories of the curve: $y = ce^{-3x}$.

11. Apply power series method to solve: $y' + xy = 0$.

12. Find inverse Laplace transform of $\frac{9}{s(s^2-9)}$.

13. Find the solution $u(x, y)$ of PDE: $u_{yy} + 16u = 0$.

14. Show that: $\cosh z = \cosh x \cos y + i \sinh x \sin y$.