KATHMANDU UNIVERSITY
End Semester Examination
March, 2026

Level : B.E.
Year : II
Time : 2 hrs. 30 mins.

Course : MATH 207
Semester : I/II
F. M. : 55

SECTION "C"
[3 Q. × 7 = 21 marks]

1. Suppose $y_1(x)$ is a known (non-trivial) solution of the homogeneous linear differential equation $y'' + p(x)y' + q(x)y = 0$. Find a second linearly independent solution $y_2(x)$, and hence obtain a fundamental set (basis) of solutions. Write the formula for the particular solution of the non-homogeneous equation $y'' + p(x)y' + q(x)y = r(x)$ using the method of variation of parameters. Using this method, solve $y'' + 2y' + y = e^{-x}\cos x$.

[3+1+3]

2. Let $F(s)$ denote the transform of the function $f(t)$ which is piecewise continuous and satisfies the growth restriction condition. Then for $s > 0, s > k$, and $k > 0$, show that $L\left(\int_0^t f(u)du\right) = \frac{F(s)}{s}$. Hence, find $L\left(\int_0^t \sin 2t\, dt\right)$.

Also, find the inverse transform of $\ln\left(\frac{s^2+1}{(s-1)^2}\right)$.

[3+2+2]

3. Evaluate $\int_C \mathrm{Im}(z^2)dz$ counterclockwise around the triangle with vertices 0, 1, i. State Independence of path. Verify independence path for the integral e^z from 0 to $1 + i$ (a) over the shortest path and (b) over the x-axis to 1 and then straight up to $1 + i$. [3+1+3]

OR

Write down the formula for the residue of $f(z)$ at $z = z_0$ of order n. State and prove Cauchy Residue Theorem. Evaluate $\oint_C \left(\frac{z\, e^{\pi z}}{z^4-16} + z^2 e^{\pi/z}\right) dz$ where C is the ellipse $9x^2 + y^2 = 9$ counterclockwise.

[1+3+3]

SECTION "D"
[6Q. × 4 = 24 marks]

4. Using $P_n(x) = \sum_{m=0}^{M} \frac{(-1)^m(2n-2m)!}{m!\,(n-m)!\,(n-2m)!} x^{n-2m}$, show that $P_2(x) = \frac{1}{2}(3x^2 - 1)$.

5. Verify that $u = xy$ is harmonic function. Determine its harmonic conjugate function $v(x, y)$ and find the corresponding analytic function $f(z)$.

P.T.O.

6. Solve the initial value problem

$$y^{iv} - 5y'' + 4y = 0;\ y(0) = 1,\ y'(0) = 0, y''(0) = 0,\ y'''(0) = 0.$$

7. Find the D'Alembert solution of $u_{xx} + u_{xy} - 2u_{yy} = 0$.

OR

Express the Laplace equation $u_{xx} + u_{yy} + u_{zz} = 0$ into cylindrical form.

8. Using variable separation method, find the solution of $u_{tt} = c^2 u_{xx}$ for separation constant $K < 0$.

9. Evaluate $\int_{-\infty}^{\infty} \frac{dx}{(1+x^2)^3}$ using contour integral.

SECTION "E"
[5Q. × 2 = 10 marks]

10. If y_1 and y_2 are the solution of $y_1' + py_1 = r_1$ and $y_2' + py_2 = r_2$ respectively with same p. What can you say about the sum $y_1 + y_2$?

11. State and Prove first shifting theorem of Laplace transform.

12. Determine the linear fractional transformation that maps the points $0,\ 1, \infty$ onto $-1, -i, 1$.

13. Find the principal value of i^{1-i}.

14. Solve $y' = (y - 2)\cot x$.