KATHMANDU UNIVERSITY
End Semester Examination
March/April, 2025

Marks Scored:

Level : B.E.		Course : MATH 207
Year : II		Semester : I
Exam Roll No. :	Time: 30 mins.	F. M. : 20
Registration No.:		Date 01 APR 2025

SECTION "A"
[10 Q. × 1 = 10 marks]

Fill in the blank space(s) by writing the most appropriate word(s) or symbol(s).

1. The differential equation $(px + qy)dx + (ax + by)dy$ will be exact if and only if __________, where a, b, p and q are constants.

2. The integrating factor of the linear differential equation $y' + 2xy = xe^x$ is __________.

3. The ordinary differential equation of the form $x^2y'' + a\,xy' + by = 0$ with given constants a and b; is known as __________.

4. The Laplace transform of $2\cos^2 \omega t =$ __________.

5. Solution of the PDE, $u_{xx} = 0$ as a function of x and y is__________.

6. The Maclaurin series $1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots$ is the series expansion of __________.

7. If $z = 2 + 3i$, then $z\,\bar{z} =$ __________, where $\bar{z}$ denotes the complex conjugate of z.

8. $f(z) = u(x, y) + i\,v(x, y)$ is analytic in a domain D if and only if the first partial derivatives of u and v satisfy __________.

9. $\oint_C \frac{1}{z^2-4}\,dz =$ __________, where C is the circle, $|z| = 1$.

10. The fixed points of the mapping $f(z) = \frac{z-1}{z+1}$ are __________.

SECTION "B"

[10 Q.× 1 = 10 marks]

Fill in the blank space(s), **DO NOT TICK**, by selecting the most appropriate answers from among the given ones.

11. The order of the differential equation formed after eliminating arbitrary constants from $y = A\sin x + B\cos x$ is ____________________.

    [1; 2; 3; not defined]

12. The orthogonal trajectories of the family of curves $y = cx$ is ________________.

    [$x = ky$; $x^2 + y^2 = k$; $y = x^2 + k$; $xy = k$]

13. The Wronskian of the two functions $y_1 = e^x,\ y_2 = xe^x$ is ________________.

    [e^x; e^{2x}; xe^{2x}; $e^{2x} - xe^{2x}$]

14. While solving differential equation $y'' + 3y' - 4y = 2e^x$ by the method of undetermined coefficients, the choice for the solution y_p is______________.

    [Ke^x; $2Ke^x$; kxe^x; $x + Ke^x$]

15. $P_n(-1) =$ _____________, where $P_n(x)$ is the Legendre polynomial of degree n.

    [-1; $(-1)^n$; $(-1)^{n+1}$; 1]

16. The Laplace transform of the function $f(t) = (t-1)\,u(t-1)$ is_______________.

    [$\frac{1}{(s-1)^2}$; $\frac{e^{-s}}{(s-1)^2}$; $\frac{e^{-s}}{s^2}$; $\frac{e^{-s}(s+1)}{s^2}$]

17. The convolution $(1 * t)$ is ________________.

    [$\frac{1}{t}$; $\frac{t}{2}$; $\frac{t^2}{2}$; t]

18. The PDE $u_{xx} + 4u_{xy} + 4u_{yy} = 0$ is________________.

    [Elliptic; Hyperbolic ; Parabolic; Circular]

19. Which one is true? _____________.

    [$\cos z = \cos x \cosh y + i \sin x \sinh y$; $\sin iz = \sin hz$; $\cos h\, iz = \cos z$; $e^{\pi i} = 1$]

20. The residue of the function $f(z) = \frac{e^z}{z}$ at $z = 0$ is _____________.

    [1; e; $\frac{e}{2!}$; $-\frac{e}{2!}$]

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Level : B.E.		Course : MATH 207
Year : II	01 APR 2025	Semester : I
Time : 2 hrs. 30 mins.		F. M. : 55

SECTION "C"
[3 Q. × 7 = 21 marks]

1. Define differential equation. If the functions p, q and r are continuous on an open interval I, and if the functions y_1 and y_2 form a basis solution of the homogeneous differential equation corresponding to the non-homogeneous equation $y'' + p(x)y' + q(x)y = r(x)$. Show that the particular solution to this equation is $y_p = -y_1 \int \frac{y_2 r}{W} dx + y_2 \int \frac{y_1 r}{W} dx$ where W is the Wronskian of y_1 and y_2. Find the particular solution of the differential equation $y'' - y = e^x$. [1+3+3]

2. State and prove the Convolution theorem of the Laplace transform. Solve the differential equation $y'' + 5y' + 4y = 2e^{-2t}, y(0) = 0, y'(0) = 0.$ [4+3]

OR

Define unit step function. If $F(s) = \frac{e^{-s}}{s^2+\pi^2} + \frac{e^{-2s}}{s^2+\pi^2} + \frac{e^{-3s}}{(s+2)^2}$, find its inverse Laplace transform. Find Laplace transform of $f(t) = \begin{cases} 0, & 0 < t < 1 \\ t^2, & 1 < t < 2. \\ 0, & t > 2 \end{cases}$ [1+3+3]

3. Define singularities and their types. Evaluate $\oint_C \frac{z^2}{(z-1)^2(z-2)} dz$ where C is $|z| = 2.5$. Find the principal value of $\int_{-\infty}^{\infty} \frac{dx}{(x^2-3x+2)(x^2+1)}$. [2+3+2]

SECTION "D"
[6Q. × 4 = 24 marks]

4. Test for the exactness and solve, $\cos(x+y)dx + (3y^2 + 2y + \cos(x+y))dy = 0$.

5. Prove that $J_{-n}(x) = (-1)^n J_n(x)$ where $J_n(x) = x^n \sum_{m=0}^{\infty} \frac{(-1)^m x^{2m}}{2^{2m+n}\, m!\Gamma(n+m+1)}$ and n is a positive integer and find the value of $J_{-1}(x) + J_1(x)$.

P.T.O.

6. Use the variable separation method to find the solution $u(x, y)$ of the partial differential equation $u_x + u_y = (x + y)u$.

OR

Reduce the equation $u_{xy} = u_{yy}$ to the canonical form and then find the general solution.

7. Show that the function $u(x, y) = 2xy - y$ is harmonic; find its harmonic conjugate $v(x, y)$ and corresponding analytic function $f(z) = u(x, y) + iv(x, y)$.

8. Develop the Laurent's series of the function $f(z) = \frac{1}{(z+1)(z+3)}$ valid for $1 < |z| < 3$.

9. An inductor of 2 henrys and a resistance of 20 ohms are connected in series combination with electromotive force (emf) E volts. If the current is zero at time t equals zero, find the current at the end of 0.5 seconds if $E = 125$ volts.

SECTION "E"

[5Q. × 2 = 10 marks]

10. Solve the Euler-Cauchy equation $x^2y'' - 5xy' + 9y = 0$.

11. Use the power series method to solve $y' = y$.

12. Find general values of $(1 + i)^{1-i}$.

13. Find the radius of convergence of the series $\sum_{n=0}^{\infty} \frac{(2n)!}{(n!)^2} (z - 3i)^n$.

14. Determine the linear fractional transformation that maps $z_1 = -1,\ z_2 = i,\ z_3 = 1$ onto $w_1 = 0, w_2 = i,\ w_3 = \infty$ respectively.