

KATHMANDU UNIVERSITY  
End Semester Examination  
March/April, 2025

Marks Scored:

Level : B.E.

Year : II

Exam Roll No. :

Time: 30 mins.

Registration No.:

Course : MATH 207

Semester : I

F. M. : 20

Date :

03 APR 2025

SECTION "A"

[10 Q. × 1 = 10 marks]

Fill in the blank space(s) by writing the most appropriate word(s) or symbol(s).

1. The degree of differential equation  $y'' = \sqrt{y' + xy}$  is \_\_\_\_\_.
2. A basis for the solutions of the ODE  $y'' - 9y = 0$  is \_\_\_\_\_.
3. The equation  $-ydx + xdy = 0$  has integrating factor \_\_\_\_\_.
4. The Legendre's polynomial of degree  $n$  is given by  
$$P_n(x) = \sum_{m=0}^M \frac{(-1)^m (2n-2m)!}{2^n m! (n-m)! (n-2m)!} x^{n-2m}$$
 where  $M = \frac{n}{2}$  or  $\frac{n-1}{2}$ , whichever is an integer, then  $P_2(x) =$  \_\_\_\_\_.
5. The Laplace transform of  $e^t t^3$  is \_\_\_\_\_.
6. If  $F'(s)$  denotes the derivative of Laplace transform with respect to  $s$ , then  $L^{-1}\{F'(s)\} =$  \_\_\_\_\_.
7. The convolution  $e^t * e^{-t}$  is \_\_\_\_\_.
8. The solution of the PDE  $u_{xx} = 0$  is \_\_\_\_\_.
9. The Laurent series of  $f(z) = \frac{1}{(1-z)z^2}$ , ( $0 < |z| < 1$ ) is \_\_\_\_\_.
10. If  $f(z)$  is analytic within and on a closed curve  $C$  and if  $z_0$  is any point inside  $C$ , then  $f(z_0) =$  \_\_\_\_\_.

## SECTION "B"

[10 Q. × 1 = 10 marks]

Fill in the blank space(s), **DO NOT TICK**, by selecting the most appropriate answers from among the given ones.

11. The Wronskian of two solutions  $y_1 = e^{2x}$  and  $y_2 = xe^{2x}$  of a second-order ODE is \_\_\_\_\_.  
 [  $e^{2x}$ ;  $xe^{2x}$   $(1+x)e^{2x}$ ;  $e^{4x}$  ]
12. A general solution of the differential equation  $x^2y'' - 5xy' + 9y = 0$  for all positive  $x$  is \_\_\_\_\_.  
 [  $y = (c_1 + c_2 \ln x)x^3$ ;  $y = (1 + \ln x)x^3$ ;  
 $y = (c_1 + c_2x)x^3$ ;  $y = (c_1 + c_2x)e^{3x}$  ]
13. For integer  $\nu = n$ ,  $(-1)^n J_{-n}(x) =$  \_\_\_\_\_, where  $J_n(x)$  is the Bessel's polynomial of order  $n$ .  
 [  $J_n(x)$ ;  $(-1)^n J_{-n}(x)$ ;  $J_n(x) + J_{-n}(x)$ ;  $J_{-n}(x)$  ]
14. The Laplace transform of  $\delta(t - 5)$  is \_\_\_\_\_.  
 [  $\frac{e^{-5s}}{s}$ ;  $\frac{e^{5s}}{5}$ ;  $e^{5s}$ ;  $e^{-5s}$  ]
15. The general solution of  $y''' + 3y'' + 3y' + y = 0$  is \_\_\_\_\_.  
 [  $(c_1 + c_2x + c_3x^2)e^x$ ;  $c_1e^x + c_2e^x + c_3e^x$ ;  
 $(c_1 + c_2x + c_3x^2)e^{-x}$ ;  $c_1e^{-x}$  ]
16. The partial differential equation  $y u_{xx} + u_{yy} = 0$  is hyperbolic if \_\_\_\_\_.  
 [  $y \neq 0$   $y < 0$   $y > 0$   $y = 0$  ]
17. The value of  $|e^{x+iy}|$  is \_\_\_\_\_.  
 [  $e^x$ ;  $\sqrt{(x^2 + y^2)}$ ;  $e^{\sqrt{(x^2 + y^2)}}$ ;  $\pi e^x$  ]
18. For the function  $f(z) = \frac{\sin z}{z^2}$  of a complex variable, the point  $z = 0$  is \_\_\_\_\_.  
 [ a pole of order 3; a pole of order 2; a pole of order 1; not a singularity ]
19. The analytic function  $f(z) = \frac{z^2 - 9}{z - 2}$  has zeros at \_\_\_\_\_.  
 [ 3 & -3; 2; 2, 3 & -3; 3i & -3i ]
20. The solution of the equation  $e^z = -2$  is given by  $z =$  \_\_\_\_\_.  
 [  $\ln 2 + 0i + 2n\pi i$ ;  $\ln 2 + \pi i + 2n\pi i$   $\ln 2 - 0i + 2n\pi i$ ;  $\ln 2 - \pi i + 2n\pi i$  ]

KATHMANDU UNIVERSITY  
End Semester Examination  
March/April, 2025

Level : B.E.  
Year : II  
Time : 2 hrs. 30 mins.

03 APR 2025

Course : MATH 207  
Semester : I  
F. M. : 55

SECTION "C"

[3 Q. × 7 = 21 marks]

1. What is meant by the solution of the differential equation? Find the general solution of  $y' + y \sin x = e^{\cos x}$ . A thermometer, reading  $5^\circ C$ , is brought into a room whose temperature is  $22^\circ C$ . One minute later the thermometer reading is  $12^\circ C$ . How long does it take until the reading is  $21.9^\circ C$ ? [1+3+3]
2. Define Laplace transform of a function  $f(t)$  defined for  $t > 0$ . Find the inverse transform of the following functions  
(a)  $\frac{1}{(s-1)(s+4)}$       (b)  $\frac{k_0(s+a)+k_1}{(s+a)^2}$       (c)  $\frac{s}{(s^2-9)^2}$       [1+2+2+2]
3. State the Cauchy–Riemann equations. Convert  $e^{1+i}$  and  $\ln(1+i)$  into  $x+iy$  form. Find the values of  $\cos(1+i)$  and  $\cosh(1+i)$ . Also, find the principal value of  $(1+i)^{1+i}$ . [1+2+2+2]

OR

State and prove Cauchy's residue theorem. Using the residue theorem, evaluate  $\int_C \frac{1}{(z-3)(z-1)^2} dz$ , where  $C$  is the circle  $|z-2|=3$  taken counterclockwise. [3+4]

SECTION "D"

[6Q. × 4 = 24 marks]

4. Prove that  $J_{-n}(x) = (-1)^n J_n(x)$ ,  $n \in Z$ , where the symbols have their usual meanings.
5. Integrate
  - a.  $\int_C \operatorname{Re} z dz$ , where  $C$  is the shortest path from  $1+i$  to  $3+3i$ .
  - b.  $\int_{-\pi i}^{\pi i} \cos z dz$
6. Using the Laplace transform, solve  $y'' - 2y' - 3y = 36e^{2x}$ ,  
 $y(0) = 9, y'(0) = 25$ .

P.T.O.

7. Classify, transform to normal form and solve:  $u_{xx} + 4u_{xy} + 4u_{yy} = 0$ .

**OR**

Find the solution of the equation  $u_x + u_y = (x + y)u$  by separating variables.

8. Show that the function  $v = xy$  is harmonic function. Also, find a corresponding analytic function  $f(z) = u(x, y) + iv(x, y)$ .
9. Evaluate  $\int_0^{2\pi} \frac{1}{5-4 \sin \theta} d\theta$ .

SECTION "E"  
[5Q.  $\times$  2 = 10 marks]

10. Use Power series method to solve the differential equation  $y'' + y = 0$
11. Find the orthogonal trajectories of  $y = ce^{-x^2}$ .
12. Find the linear fractional transformation which maps the points  $\infty, 1, 0$  onto  $0, 1, 0$  respectively.
13. Find the center and radius of convergence of the power series  $\sum_{n=1}^{\infty} \frac{(z-i)^n}{n^2}$ .
14. Solve  $yy' = \frac{y^2 - x^2}{2x}$ .