Marks Scored:

KATHMANDU UNIVERSITY
End Semester Examination
March/April, 2017

Level : B.E./B. Sc./B. Tech.
Year : II
Exam Roll No. :

Registration No. :

Time : 30 mins.

Course : MATH 207
Semester : I & II
F. M. : 20
Date : MAR 2 4 2017

SECTION "A"
[10 Q. × 1 = 10 marks]

Fill in the blank space(s) by writing most appropriate word(s) or symbol(s):

1. The principal value $Arg\, z$ of the complex number $z = -1$ is ________________.

2. A second order partial differential equation $Au_{xx} + 2Bu_{xy} + Cu_{yy} = F(x, y, u, u_x, u_y)$ is a parabolic if ________________.

3. If $f(t) = \begin{cases} k, & 0 < t < b \\ 0, & t > b \end{cases}$. Then the Laplace transform of $f(t)$ is ________________.

4. For an integer $n > 0$, the Bessel functions $J_n(x)$ and $J_{-n}(x)$ are linearly dependent, and are related with ________________.

5. The function $y = ce^x$, where c is an arbitrary constant, is a solution of the first order differential equation ________________.

6. $\oint_C \frac{dz}{z-a} =$ ________________, where $C: |z-a| = \rho$ (Clockwise)

7. The Wronskian of the functions $y_1 = x$, $y_2 = x^2$ and $y_3 = x^3$ is ________________.

8. The general solution $u(x, y)$ of the partial differential equation $u_x = u$ is ________________.

9. An equation of the form $x^2y'' + axy' + by = 0, \quad x \neq 0$ is called a(n) ________________.

10. If a complex function $f(z)$ is analytic everywhere on a simply connected domain D, and C is a simple closed curve in D, then $\oint_C f(z)dz =$ ________________.

SECTION "B"

[10 Q × 1 = 10 marks]

Fill in the blank space(s) by selecting the most appropriate answer from among the given ones:

11. If $u(t-a)$ is a unit step function, then $L\{u(t-a)\}=$ ______________.

[e^{-as}; e^{as}; $\frac{e^{-as}}{s}$; $\frac{e^{as}}{s}$]

12. The only solution $u(r)$ of the Laplace equation $u_{rr}+\frac{1}{r}u_r=0$ depending only on r is given by ______________, where a and b are arbitrary constants.

[ar; $ar+b$; $a\log r$; $a\log r+b$]

13. Let $P_n(x)$ be a Legendre's polynomial. Then the expression $P_n(x)=\frac{1}{2^n n!}\frac{d^n}{dx^n}\left(x^2-1\right)^n$, $n=0,1,2,\cdots$ is called a(n) ______________ formula.

[Rodrigue; Euler; Bessel; Taylor]

14. The function ______________ is an integrating factor of $\sin y\,dx+\cos y\,dy=0$.

[x; xy; e^{-x}; e^{x}]

15. If z is a complex variable, then $\cosh^2 z-\sinh^2 z=$ ______________.

[-1; 0; 1; π]

16. $L^{-1}\left\{\frac{1}{s(s-\pi)}\right\}=$ ______________.

[$\frac{1}{\pi}\left(1-e^{\pi t}\right)$; $\frac{1}{\pi}\left(1-e^{-\pi t}\right)$; $\frac{1}{\pi}\left(e^{\pi t}-1\right)$; $\frac{1}{\pi}\left(e^{-\pi t}-1\right)$]

17. The orthogonal trajectories of the curve $xy=c$ is ______________, where c and λ are constants.

[$xy=\lambda$; $x^2+y=\lambda$; $xy=\lambda+cx$; $x+y^2=\lambda$]

18. The residue of the function $\frac{\sin z}{z^4}$ at $z=0$ is ______________.

[$-\frac{1}{6}$; $-\frac{1}{3}$; $\frac{1}{6}$; $\frac{1}{3}$]

19. The partial differential equation $u_{xx}+xu_{yy}$ is elliptic if ______________.

[$x=0$; $x<0$; $x>0$; $x=-1$]

20. $\int_C \bar{z}\,dz=$ ______________, where C: from origin along the parabola $y=x^2$ to $1+i$.

[$\frac{i}{3}$; $\frac{1}{3}$; $1+\frac{i}{3}$; $1-\frac{i}{3}$]

KATHMANDU UNIVERSITY

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Time : 2 hrs. 30 mins.

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F. M. : 55 MAR 2 4 2017

SECTION "C"

[3 Q. × 7 = 21 marks]

1. State Cauchy integral theorem. Use this theorem to prove that if $f(z)$ is analytic in a simply connected domain D, then the integral of $f(z)$ is independent of path. Evaluate the integral $\oint_C \frac{dz}{z^2+1}$, $C : |z+i| = 1$ (Counterclockwise). [1 + 3 + 3]

OR

State Laurent theorem. Expand $\frac{1}{z^2-z}$ in a Laurent series valid for the domains $(a)\, 0 < |z| < 1$ and $(a)\, 0 < |z-1| < 1$. [2 + 5]

2. Let a, b be constants, and $L(y) = y''(x) + a\,y'(x) + b\,y(x)$ and let λ_1, λ_2 be the roots of the characteristic equation $P(\lambda) = \lambda^2 + a\lambda + b = 0$. If λ_1 and λ_2 are real and unequal, then show that $y_1(x) = e^{\lambda_1 x}$ $y_2(x) = e^{\lambda_2 x}$ and $c_1 y_1(x) + c_2 y_2(x)$ where c_1, c_2 are arbitrary constants, are the solutions of $L(y) = 0$. Also, solve the initial value problem $4y''(x) + 20y'(x) + 16y(x) = 0$, $y(0) = 1$, $y'(0) = 2$. [3 + 4]

3. Find the d'Alembert's solution $u(x,t)$ of the one dimensional wave equation $u_{tt} = c^2 u_{xx}$, $0 < x < L, t > 0$ subjected to the initial conditions $u(x,0) = f(x)$, $u_t(x,0) = 0$ and the boundary conditions $u(0,t) = 0$, $u(L,t) = 0$. [4 + 3]

SECTION "D"

[6 Q × 4 = 24 marks]

4. Show that $\frac{d}{dx}\left[x^{\gamma} J_{\gamma}(x)\right] = x^{\gamma} J_{\gamma-1}(x)$, where the symbols have their usual meanings.

5. Solve $y''(t) + 10y'(t) + 24y(t) = 144$, $y(0) = 0$, $y'(0) = 12$ using the Laplace transform method.

6. Use contour integration method to evaluate the integral $\int_{-\infty}^{\infty} \frac{x+2}{x^3+x}\,dx$

OR

Evaluate the integral $\int_0^{2\pi} \frac{d\theta}{2+\cos\theta}$ using contour integration method.

7. Show that the equation $2xy\,dx + x^2 dy = 0$ is exact, and solve it.

8. Show that $u(x, y) = e^{-x}(x\sin y - y\cos y)$ is harmonic, and hence find its conjugate harmonic function.

9. Find the general solution of the differential equation $y''(x) - 4y'(x) + 4y(x) = x^2 e^x$ by variation of parameters method.

SECTION "E"

[5 Q × 2 = 10 marks]

10. Find the solution $u(x, y)$ of the partial differential equation $u_y + 2yu = 0$.

11. Prove that $\cosh iz = \cos z$.

12. Find $L\{t^5 e^t\}$.

13. Evaluate $\oint_C \frac{dz}{z-\pi}$, $C: |z - 4\pi| = \pi$ (Counterclockwise).

14. Verify that $x^2 + y^2 = 1$ is the solution of $x + yy' = 0$.