Marks Scored:

KATHMANDU UNIVERSITY
End Semester Examination [C]
June, 2018

Level : B.E/B.Sc./B.Tech
Year : II/II

Course : MATH 207
Semester : I/II

Exam Roll No .:

Time : 30 mins.

F.M. : 20

Registration No .:

Date JUN 13 2018

SECTION "A"
[10 Q. × 1 = 10 marks]

Fill in the blank space(s) by writing most appropriate word(s) or symbol(s).

1. A first order differential equation of the form $g(y)y' = f(x)$ is called.....

2. Laplace transform is a .....operator

3. Legendre's polynomial of degree 2, $P_2(x)$ =.....

4. $L(e^{3t}cos5t)$ = ...

5. Zeros of order k of analytic function $f(z)$ = poles of order k of analytic function.....

6. $\oint_C e^z dz$ = ...

7. If $f(z)$ is analytic in a domain D, then value of first derivative at a point $Z = Z_0$ in D are given by the formula $f'(Z_0)$ = ...

8. The radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{2n}{n!}(z+i)^{2n-1}$ =.....

9. If $f(t)$ and $g(t)$ are the analytic functions then their convolution is $f(t) * g(t)$ =.....

10. $L(f'''(t))$ = ...

SECTION "B"

[10 Q. × 1 = 10 marks]

Fill in the blank space(s), DO NOT TICK, by choosing the most appropriate answer from among the given ones.

11. The integrating factor of $siny\,dx + cosy\,dy = 0$ is .....
[*tany;* *coty;* *tanx;* e^x]

12. $f(z) = \frac{e^z}{z(z-2)(z-4)^3}$ has poles of orders .....and.....
[1*and* 0; 0, 1 *and* 2; 1 *and* 2; 1 *and* 3]

13. If w denotes the wronskian then $w(x^3, x^2 lnx^2)$ =.....
[$2x^4 - x^4 lnx^2$, $2x^4 + x^4 lnx^2$, $x^4 lnx^2 - 2x^4$, $x^7 lnx$;]

14. $L\{\delta(t-a)\}$ =.....
[e^{-as}; $\frac{e^{-as}}{s}$; $\frac{n!}{s^{n+1}}$; $\frac{1}{s^2}$]

15. The solution of PDE $u_{xx} = u_x$ is .....
[$Axe^y + c$; $Aye^{-y} + c$; $Axe^{-x} + c$; $Ae^{xy} + c$]

16. If z is a complex variable then $|sinz|^2$ =.....
[$cos^2x + sinh^2y$; $cos^2x + cosh^2y$ $sin^2x + cosh^2y$; $sin^2x + sinh^2y$]

17. $e^{\frac{\pi}{2}i}$ =.....
[1; i; -1; -i]

18. The function $f(z) = 3z^2 + 3i$ at $z = 2 + i$ has (u, v) as .....
[(15,9) , (15,9); (9,15); (-15,-9)]

19. If $J_n(x) = x^n \sum_{m=0}^{\infty} \frac{(-1)^m x^{2m}}{2^{2m+n} m!(n+m)!}$ is called ----------
[Euler function, Bessel function, Legendre's polynomial, Gamma function]

20. The radius of convergence of the series $\sum_{m=0}^{\infty} \frac{(-1)^m x^{3m}}{8^m}$ = ...

[$\frac{1}{8}$; 64; 8; 1]

KATHMANDU UNIVERSITY
End Semester Examination [C]
June, 2018

JUN 13 2018

Level : B.E/B.Sc./B.Tech
Year : II/II
Time : 2 hrs.30 mins.

Course : MATH 207
Semester : I/II
F.M. : 55

SECTION "C"
[3 Q. × 7 = 21 marks]

1. State Euler –Cauchy differential equation and show that Euler-Cauchy differential equation has the auxiliary equation of the form $m^2 + (a-1)m + b = 0$.Also find the general solution of the differential equation: $x^2y'' - 2xy' + 2y = 0$ [1+3+3]

2. What are the condition for a function $f(z)$ to be analytic in a domain D .Show that the derivative of analytic function at a point $z = z_0$ in a domain D is given by $f^{(n)}(z_0) = \frac{n!}{2\pi i}\oint_c \frac{f(z)}{(z-z_0)^{n+1}}dz$. By using this formula evaluate $\oint_c \frac{z^3-2iz^2}{(z-i)^4}dz$ at $z = i$

3. Discuss harmonic and conjugate harmonic functions. For the analytic function $f(z) = e^{-x}(cosy - isiny)$ show that the function u is the harmonic, also find conjugate harmonic function of u and corresponding analytic function.

OR

Define unit step function and find its Laplace transform. Solve the initial value problem $y'' + 7y' + 12y = 21\,e^{3t},\ y(0) = 3.5, y'(0) = -10$ using Laplace transform. [1+2+4]

SECTION "D"
[6 Q. × 4 = 24 marks]

4. Solve: $y^{iv} - 9y'' - 400y = 0$ with $y(0)=0, y'(0) = 0, y''(0) = 41, y'''(0) = 0,$

5. Establish the relation between Bessel's functions $J_n(x)$ and $J_{-n}(x)$ as
$$J_n(x) = (-1)^n J_n(x)$$

6. Evaluate following real integral by using Residue theorem in counter-clockwise sense around unit circle. [4]
$$\int_0^{2\pi} \frac{d\theta}{25 - 24\,cos\theta}$$

OR

By the method of contour integration method evaluate $\int_{-\infty}^{\infty} \frac{dx}{(x^2+1)(x^2+9)}$

7. What does p.d.e: $u_{xx} - 2u_{xy} + u_{yy} = 0$ represent? Find the transformation factors for this p.d.e [1+3]

8. For the complex number z show that
$(a)\ cosz = cosx\ coshy - i\ sinx\ sinhy\ \ (b)\ \ |cosz|^2 = cos^2x + sinh^2y$ [3+1]

9. Solve the following differential equation by the method of variation of parameter : $y'' + y' - y = -6x^3 + 3x^2 + 6x$

SECTION "E"

[5 Q. × 2 = 10 marks]

10. Find by the method of convolution $L^{-1}\{\frac{1}{s(s^2+4)}\}$.

11. Solve the differential equation :$(4x^2D^2 + 12xD + 3)y = 0$.

12. Find the residue at poles of $f(z) = \frac{1-4z+6z^2}{(z^2+\frac{1}{4})(2-z)}$, C: $|z| = 1$

13. Find general power of $(2i)^i$.

14. Find the principal value $Ln(z) when\ z = 2 - 2i$