KATHMANDU UNIVERSITY
End Semester Examination [C]
June/July 2024

Marks Scored:

Level : B.E./B.Sc.
Year : II

Course : MATH 207
Semester : II

Exam Roll No. :

Time: 30 mins.

F. M. : 20

Date : 01 JUL 2024

Registration No.:

SECTION "A"
[10Q. × 1 = 10 marks]

Fill in the blank space(s) by writing the most appropriate word(s) or symbol(s).

1. The Laplace transform of e^{2t} is ____________________.

2. The order of the differential equation $\left[\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2\right]^{1/2} = \left(\frac{d^3y}{dx^3}\right)^2$ is ____________________.

3. Two functions $y_1(x)$ and $y_2(x)$ are ____________ __________ if their Wronskian is not equal to zero for all x in their common domain.

4. The type of the partial differential equation $u_{xx} + 9\,u_{yy} = 0$ is ____________________.

5. The Rodrigues' formula for Legendre's polynomial $P_n(x) = \alpha \frac{d^n}{dx^n}(x^2-1)^n$ where $\alpha =$ ________________.

6. $\int_C \frac{z}{z+1.5}\,dz =$ ____________________ when C is $|z| = 0.5$ (Counterclockwise).

7. If $f(z) = \sin z$, then the derivative of $f(z)$ is ____________________.

8. The complex function $f(z) = \frac{z}{(z-1)^2(z+3)}$ has a pole of order ____________________ at $z = 1$.

9. The general solution of the differential equation $y'' + 4y' = 0$ is $y(x) =$ ________________________________, where a and b are constants.

10. If $f(z)$ is analytic in a simply connected domain D, and C is a simple closed curve that lies entirely within D, then $\int_C f(z)dz =$ ____________________.

SECTION "B"
[10 Q. × 1 = 10 Marks]

Fill in the blank space(s), **DO NOT TICK**, by choosing the most appropriate answers from among the given ones.

11. If $\mathcal{L}(f(t)) = F(s)$, then $\mathcal{L}(e^{-3t} f(t)) =$ ____________________________.
[$e^{3s}F(x)$; $e^{-3s}F(s)$; $F(s-3)$; $XF(s+3)$]

12. The solution of the differential equation $y' + \frac{1}{x} = 0$ is $y(x) =$ ________________ where c is a constant.
[$\ln\frac{c}{x}$; $e^{c/x}$; $\frac{c}{x}$; $\ln\frac{c}{x^2}$]

13. The differential equation $y' + p(x)y = q(x)y^{5/2}$ is a(n) ________________ equation.
[Laplace; Potential; Bernoulli; Euler]

14. Given $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$, $0 < x < L$ subjected to initial conditions $u(x,0) = f(x)$, and $\frac{\partial}{\partial t} u(x,t)\Big|_{t=0} = 0$, then the solution $u(x,t) =$ ________________.
[$f(x+t) + f(x-t)$; $f(x+t) - f(x-t)$;
$\frac{1}{2}[f(x+t) + f(x-t)]$; $\frac{1}{2}[f(x+t) - f(x-t)]$]

15. Let $P_n(x)$ represents the Legendre polynomial of degree n, then $P_0(x) + P_2(x) =$ ________________.
[$\frac{1}{2}(3x^2 - 1)$; $\frac{1}{2}(3x^2 + 1)$; $1 + x$; $1 + \frac{x^2}{2}$]

16. $\int_C \frac{z}{z^2+1}\, dz$ where C is $|z| = \frac{1}{2}$ is ________________
[0; $\pi\, i$; $2\,\pi\, i$; $4\,\pi\, i$]

17. The complex function $f(z) = \bar{z}$ is analytic ________________.
[nowhere; only at $z = 0$; everywhere; only at $z = 1$]

18. The Principal value of $\sin h^{-1}(i)$ is ________________.
[$\frac{i\,\pi}{2}$; $i\frac{\pi}{4}$; $i\,\pi$; $n\,\pi$]

19. The PDE $u_t = u_{xx}$ is a(n) ________________ equation.
[wave; Laplace; heat; potential]

20. The general solution of $\frac{dy}{dx} = 2x\, e^{x^2 - y}$ is ________________ where c is a constant.
[$e^{x^2-y} = c$; $e^{-y} + e^{x^2} = c$; $e^{y} = e^{x^2} + c$; $e^{y} = e^{x^2} + c$]

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End Semester Examination [C]
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Level : B.E./B.Sc.	01 JUL 2024	Course : MATH 207
Year : II		Semester : II
Time : 2 hrs. 30mins.		F. M. : 55

SECTION "C"
[3 Q. × 7 = 21 marks]

1. State and prove the Cauchy-Residue Theorem. Use this theorem to evaluate the integral $\oint_C \frac{z+1}{z^4-2z^3}\,dz, \quad C: |z-1| = 2$ (Counterclockwise). [4 + 3]

OR

State and prove necessary conditions for Cauchy-Riemann equations. Derive the polar form of Cauchy-Riemann equations. [4+3=7]

2. Discuss to find the general solution of the Euler-Cauchy differential equation $x^2y'' + a\,x\,y' + by = 0$ where a and b are constants. Solve the initial value problem $x^2y'' + x\,y' + 9\,y = 0,\ y(1) = 0,\ y'(1) = 2.5.$ [4+3=7]

3. Define the Laplace transform. State and prove the first shifting theorem. Use this theorem to find the Laplace transform of $e^{at}\cos bt$ where a and b are constants. Also, find the inverse Laplace transform of $\frac{3s-137}{s^2+2s+401}$. [1+2+2+2=7]

SECTION "D"
[6 Q. × 4 = 24 Marks]

4. Find the type, normal form, and solve the partial differential equation $u_{xx} + 5\,u_{xy} + 4\,u_{yy} = 0.$

OR

Find the D'Alembert's solution $u(x,t)$ of $u_{tt} = c^2\,u_{xx}$.

5. Show that $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\sin x$, where the symbol has its usual meaning.

6. Find the Laurent series of $z^2e^{1/z}$ with center 0.

7. Derive the model equation for an RLC circuit, and use it to find the current at time t in an RLC circuit given that $R = 4$ ohms, $L = 0.5$ henrys, $C = 0.1$ farads, and $E(t) = 500\sin 2t$ volts.

P.T.O.

8. Solve the Bernoulli equation $\frac{dy}{dx} = a\,y - b\,y^2$ where a and b are constants.

9. Solve the equation $y'' + 3y' + 2.25\,y = -10\,e^{-1.5\,x}$, $y(0) = 1,\ \ y'(0) = 0$.

SECTION "E"

[5 Q. × 2 = 10 Marks]

10. Solve the ODE $y' = (x+1)e^{-x}y^2$.

11. Find the inverse Laplace transform of $\frac{1}{s^2-2s+3}$.

12. Find the radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{n^n}{n!} n^n (z - \pi\, i)^n$.

13. Solve the differential equation $y' = y$ using the power series method.

14. Find the solution $u(x, y)$ of the partial differential equation $u_{xy} = 0$ subjected to $u(x, 0) = \sin x$ and $u(0, y) = e^y$.