KATHMANDU UNIVERSITY
End Semester Examination
July/August, 2024

Level : B.E./B.Sc./B.Tech.
Year : II
Time : 2 hrs. 30mins.

Course : MATH 207
Semester : I
F. M. : 55

20 AUG 2024

SECTION "C"
[3 Q. × 7 = 21 marks]

1. When a differential equation is said to be exact? State the conditions of exactness for the first order differential equation of the form $P(x,y)dx + Q(x,y)dy = 0$. How would you convert non-exact such differential equation into exact form? Find the integrating factor of the differential equation $(3xy + y^2)dx + (x^2 + xy)dy = 0$ and solve it. [3+4]

2. Write the standard form of linear second order ordinary differential equations (ODEs). How would you classify it as homogeneous and non-homogeneous type? Write two methods to solve the second order non-homogeneous linear ODEs and explain the solution procedures briefly. Find the particular solution of the differential equation $y'' - y' - 6y = 20e^{2x}, y(0) = 0, y'(0) = 6$. [3+4]

OR

Define Laplace transform of a real valued function $f(t), t > 0$. Using definition find the Laplace transform of the function $f(t) = t^a$ for any positive real number a. Apply Laplace transform to solve the ODE $y'' + 9y = \delta(t-1),\ y(0) = 0, y'(0) = 0$, where $\delta(\cdot)$ denotes the Dirac-delta function. [1+3+3]

3. Define the derivative of a complex variable function $w = f(z)$ at a point $z = z_0$. Derive the Cauchy-Riemann equations for an analytic function $f(z) = u(x,y) + iv(x,y)$ at a point $z_0 = x_0 + iy_0$. Show that the function $f(z) = \cos x \cosh y - i \sin x \sin hy$ satisfies Cauchy-Riemann equations everywhere and also find its derivative $f'(z)$. [1+3+3]

SECTION "D"
[6 Q. × 4 = 24 marks]

4. In LC-circuit, find the current when inductance is 0.2 Henry, Capacitance is 0.005 Farad, and electromotive force is t^2 Volts, assuming the initial current and charge is zero.

P.T.O.

5. Prove that $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}} \sin x$, where $J_\nu(x) = x^\nu \sum_{m=0}^{\infty} \frac{(-1)^m x^{2m}}{2^{2m+\nu} m!\, \Gamma(\nu+m+1)}$ is the Bessel's function of first kind of order ν.

6. Express the function $f(x) = \begin{cases} 1 - e^{-t}, & 0 < t < \pi \\ 0, & \text{else where} \end{cases}$ into unit step function form and then find its Laplace transform.

7. Classify the type of the partial differential equation $u_{xx} + u_{xy} - 2u_{yy} = 0$. Transform this equation into normal form and solve it.

8. Evaluate the integral $\oint_C \frac{(3z+2)^2}{z(z-1)(2z+5)} dz$, where C be the circle $|z| = 2$ oriented counter-clockwise

OR

Evaluate the real integration $\int_{-\infty}^{\infty} \frac{dx}{(1+x^2)^2}$ using the complex integration theory.

9. Find the linear fractional transformation that maps the points $i, 1, -i$ onto $1, 0, -1$.

SECTION "E"
[5 Q. × 2 = 10 marks]

10. Find the power series solution of the first order differential equation $y' = y$.

11. Find the inverse Laplace of $F(s) = \frac{1}{s^2(s^2+4)}$.

12. For what value of c the function $u = e^{-\pi^2 t} \sin 4x$ satisfies the 1-dimensional heat equation $u_t = c^2 u_{xx}$.

13. Find center and radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{(2n)!}{(n!)^2} (z + i)^n$.

14. Find the all possible values and the principle value of the general power i^{-i}.

KATHMANDU UNIVERSITY
End Semester Examination
July/August, 2024

Marks Scored:

Level : B.E./B.Sc./B.Tech.
Year : II

Course : MATH 207
Semester : I
F. M. : 20

Exam Roll No. :

Time: 30 mins.

Registration No.:

Date : 20 AUG 2024

SECTION "A"

[10 Q. × 1 = 10 marks]

Fill in the blank space(s) by most appropriate words or symbol(s):

1. The degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^{\frac{3}{2}} = \left(\frac{dy}{dx}\right)^2 + x^2y^3$ is ....
2. The Wronskian of the function $y_1 = \sin x$ and $y_2 = \cos x$ is ....
3. The differential equation that model the flow of current $I = I(t)$ at time t in the LC-circuit with time varying electromotive force $E(t)$ Volts, capacitor of Capacitance C Farad, and inductor of Inductance L Henry is given by ....
4. For an integer n, the Bessel's functions $J_n(x)$ and J_{-n} are related by the equation ....
5. The inverse Laplace transform of $F(s) = \frac{1}{s^2+9}$ is ....
6. The Laplace transform of the Dirac delta function $\delta(t-a)$ is ....
7. The second order partial equation $u_{xx} + 2u_{xy} + u_{yy} = 0$ is .... type in PDE classification.
8. The principal argument $\operatorname{Arg} z$ of the complex number $z = a - ia, a > 0$ is ....
9. If $f(z) = u(x,y) + iv(x,y)$ is analytic function in a domain D, then u and v satisfy Cauchy-Riemann equations .... and .... in the domain D.
10. The residue of $f(z) = \frac{4z}{z^2+1}$ at the singular point $z = i$ is ....

SECTION "B"

[10 Q. × 1 = 10 marks]

Fill in the blank space(s), ***DO NOT TICK,*** *by selecting the most appropriate answers from among the given ones.*

11. The integrating factor of the differential equation $\frac{dy}{dx} + p(x)y = r(x)$ is ....

[$e^{\int r(x)dx}$; $\int r(x)\,dx$; $e^{\int p(x)dx}$; $\int p(x)\,dx$]

12. The general solution of the differential equation $\frac{dy}{dx} = -y$ is ....

[$y = Ce^{-x}$; $y = Ce^{x}$; $y = x + c$; $y = \ln x + c$]

13. The equation........................is the characteristic equation of the differential equation $y'' + y' - 6y = 0$.

[$\lambda^2 + \lambda + 6 = 0$; $\lambda^2 + \lambda - 6 = 0$; $\lambda^2 - \lambda - 6 = 0$; $\lambda^2 - \lambda + 6 = 0$]

14. The second order differential equation of the form $(1 - x^2)y'' - 2xy' + n(n-1)y = 0$ for any real constant n is known as........................equation

[Bessel's; Bernoulli's; Euler-Cauchy; Legendre's]

15. The convolution $(1 * 1) =$ ....

[1; $\frac{t}{2}$; t; $\frac{t^2}{2}$]

16. The partial differential equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ is known as 1-dimensional ....equation.

[heat; Poisson; Laplace; wave]

17. The parametric representation of line segment joining from $z = 0$ to $z = 1 + i$ is ...., $0 \le t \le 1$.

[$z(t) = -t - it$; $z(t) = t + it$; $z(t) = t - it$; $z(t) = -t + it$]

18. The value of the complex integral $\oint_C e^z dz =$ ...., where C is unit circle oriented positively.

[0; 1; $2\pi i$; ∞]

19. The Taylor's series $\sum_{n=0}^{\infty} z^n$ for all $|z| < 1$ converges to the function $f(z) =$....................

[$\frac{1}{1+z}$; $\frac{1}{1-z}$; $\frac{1}{z}$; e^z]

20. The function $f(z) = \frac{z^2-1}{z^2(z-i)}$ has simple pole at $z =$...........................

[0; 1; i; -1]