KATHMANDU UNIVERSITY
End Semester Examination
June/July, 2023

Marks Scored:

Level : B.E./B.Sc./B.Tech.
Year : II

Course : MATH 207
Semester : I & II

Exam Roll No. :

Time: 30 mins.

F. M. : 20

Date : 16 JUL 2023

Registration No.:

SECTION "A"

[10Q. × 1 = 10 marks]

Fill in the blank space(s) by writing most appropriate word(s) or symbol(s).

1. Integrating factor of the ordinary differential equation $\frac{dy}{dx} + y = x$ is ______________.

2. A curve that intersects each member of a given family of curves at right angles is called a/an ______________________________ of the family.

3. The Wronskian of the solutions $y_1 = e^{-x}$ and $y_2 = e^{x}$ of the differential equation is $y'' - y = 0$ is ____________________.

4. The value of $|e^{ix}|$ is ________________, where the symbols have their usual meanings.

5. The Laplace transform of the function $e^{\alpha t} \sin \beta t$ is ____________________.

6. The solution $u(x, y)$ of $u_x = u$ is ______________.

7. Maclaurin series expansion for the function $f(x) =$ ____________ is $1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$.

8. The analytic function $f(z) = z^2$ has a zero of order ________________ at $z = 0$.

9. The radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{z^n}{n}$ is $R =$ __________.

10. The mapping $f(z) = \frac{1}{z}$ has two fixed points______________.

SECTION "B"

[10 Q. × 1 = 10 marks]

Fill in the blank space(s), DO NOT TICK, by selecting the most appropriate answers from among the given ones.

11. If the differential equation $\alpha y\, dx + \beta x\, dy = 0$ is exact, then __________
[$\alpha = \beta$; $\alpha + \beta = 0$; $\frac{1}{\alpha}(\alpha - \beta)$; $\frac{1}{\beta}(\alpha - \beta)$]

12. While solving differential equation $y'' + y' = \cos x$ by the method of undetermined coefficients, the choice for the particular solution y_p is_______________.
[$A \sin x$; $B \cos x$; $A \sin x + B \cos x$; $AB \sin x \cos x$]

13. If $P_n(x)$ is a Legendre function of degree n, then $P_2(x) =$ ______________.
[x^2; $\frac{1}{2}(3x^2-1)$; $\frac{1}{2}(3x^2-2)$; $\frac{1}{2}(x^2+1)$]

14. For any integer n, $J_{-n}(x) =$ ______________, where the symbols have their usual meanings.
[$-J_n(x)$; $J_n(x)$; $(-1)^n J_n(x)$; $J_n(-x)$]

15. If the Laplace transform of the function $f(t),\ t \geq 0$ is $L\{f\}$, then the Laplace transform of its second order derivative $f''(t)$ is $L\{f''(t)\} =$ ________________.
[$s^2L\{f\} - sL\{f\} - L\{f\}$; $s^2L\{f\} - sf(0) - f'(0)$;
$s^2L\{f\} - sf'(0) - f''(0)$; $s^2L\{f\} - f(0) - sf'(0)$]

16. The inverse Laplace transform of $\frac{e^{-2s}}{s^2+1}$ is ________________.
[$\sin(t-2)\,u(t-2)$; $\cos(t-2)u(t-2)$;
$\sinh(t-2)u(t-2)$; $\cosh(t-2)u(t-2)$]

17. The Laplace equation $u_{xx} + u_{yy} = 0$ is________________.
[Elliptic; Hyperbolic ; Parabolic; Circular]

18. Which one of the following is **NOT TRUE**? ______________.
[$e^{i\theta} = \cos\theta + i\, sin\theta$; $\sin z = \frac{1}{2}(e^{iz} - e^{-iz})$;
$\sin hz = \frac{1}{2}(e^{z} - e^{-z})$; $\cos h\, iz = \cos z$]

19. The function $f(z) = \sin\frac{1}{z}$ has the singularity $z = 0$ called ______________.
[Essential Singularity; Pole;
Non-isolated Singularity; Removable Singularity]

20. All points at which the mapping $f(z) = \cos\pi z$ is not conformal are ________________.
[$0, \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \cdots$; $0, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{5}{2}, \cdots$;
$0, \pm\pi, \pm2\pi, \pm3\pi, \cdots$; $0, \pm1, \pm2, \pm3, \cdots$]

KATHMANDU UNIVERSITY
End Semester Examination
June/July, 2023

16 JUL 2023

Level : B.E./B.Sc./B.Tech.
Year : II
Time : 2 hrs. 30 mins.

Course : MATH 207
Semester : I & II
F. M. : 55

SECTION "C"
[3Q. × 7 = 21 marks]

1. Define the order of a differential equation. Let $y_1,\ y_2$ be two linearly independent solutions of the second order differential equation $y'' + p(x)\,y' + q(x)\,y = 0$ on an open interval I. Then, show that the non-homogeneous differential equation

$$y'' + p(x)\,y' + q(x)\,y = r(x)$$

has a particular solution $y_p(x)$ given by $y_p(x) = -y_1 \int \frac{y_2 r}{W} dx + y_2 \int \frac{y_1 r}{W} dx$, where W is the Wronskian of y_1 and y_2. Find the particular solution of $y'' - 2y' + y = \frac{e^x}{x}$. [1+3+3]

2. State and prove the Convolution theorem of the Laplace transform. Solve the differential equation $y'' - y = e^{2t}, y(0) = 0, y'(0) = 1$ using the Laplace transform method. [4+3]

OR

State and prove the first and the second shifting theorems of Laplace transforms.

a. Find the inverse transform of $F(s) = \frac{2(s+1)+3}{(s+1)^2}$.

b. Find the Laplace transform of $f(t) = e^{-t}\ (0 < t < \pi)$. [2+2+1.5+1.5]

3. Define the essential singularity and pole of a complex-valued function $f(z)$. Evaluate $\oint_C \frac{50z}{(z-1)^2(z+4)} dz$ where C is a circle $|z| = 5$ (counterclockwise). Prove that $\int_0^{2\pi} \frac{d\theta}{5 - 3\cos\theta} = \frac{\pi}{2}$ using the residue integration method. [2+3+2]

SECTION "D"
[6Q. × 4 = 24 marks]

4. Prove that $\frac{d}{dx}\left[x^\nu J_\nu(x)\right] = x^\nu J_{\nu-1}(x)$, where $J_\nu(x) = x^\nu \sum_{m=0}^{\infty} \frac{(-1)^m x^{2m}}{2^{2m+\nu}\, m!\, \Gamma(\nu+m+1)}$.

5. Solve the initial value problem $y'' + y = 0.001x^2,\ \ y(0) = 0,\ y'(0) = 1.5$ by the method of undetermined coefficients.

6. Use the variable separation method to find the solution $u(x, y)$ of the partial differential equation $u_x + u_y = (x + y)u$.

OR

Express the Laplace equation $u_{xx} + u_{yy} + u_{zz} = 0$ in cylindrical coordinates.

7. Show that the complex valued function $u(x, y) = x^2 - y^2 - y$ is harmonic and find its harmonic conjugate.

8. Prove that: $\oint_C (z - z_0)^m dz = \begin{cases} 2\pi i & (m = -1) \\ 0 & (m \neq -1 \text{ and integer}) \end{cases}$, where m is the integer, z_0 a constant and C is a circle $|z - z_0| = \rho$ (counterclockwise).

9. An inductor of 2 henrys and a resistance of 20 ohms are connected in series combination with electromotive force (emf) E volts. If the current is zero at time t equals zero, find the current at the end of 0.5 seconds if $E = 125$ volts.

SECTION "E"

[5Q. × 2 = 10 marks]

10. Use the power series method to solve $y' = y$.

11. Solve Euler Cauchy equation $x^2y'' + 2xy' - 12y = 0$.

12. Find general values of $(2i)^{2i}$.

13. Expand $f(z) = \frac{1}{1-z}$ in Taylor series with center at $z_0 = 2i$.

14. Determine the linear fractional transformation that maps $z_1 = 0,\ z_2 = 1,\ z_3 = \infty$ onto $w_1 = -1, w_2 = -i,\ w_3 = 1$ respectively.