KATHMANDU UNIVERSITY
End Semester Examination [C]
January, 2018

JAN 14 2018

Level : B.E./B. Sc.
Year : II
Time : 2 hrs. 30 mins.

Course : MATH 207
Semester: II
F.M. : 55

SECTION "C"
[3Q.× 7=21 marks]

Q1. If the functions p, q and r are continuous on an open interval I, and if the functions y_1 and y_2 form a basis solution of the homogenous equation corresponding to non-homogenous equation

$$y'' + p(x)y' + q(x)y = r(x).$$

Show that the particular solution of non-homogenous equation is

$y_p = -y_1 \int \frac{y_2 r}{W} dx + y_2 \int \frac{y_1 r}{W} dx$, where W is the Wronskian of y_1 and y_2.

Use this method to find the particular solution of $y'' + 4y = 3 \operatorname{cosec} x$. [4+3]

Q2. Define Laplace transform of a function. Find the inverse transform of the following functions

(a) $\ln\left(1 + \frac{\omega^2}{s^2}\right)$ (b) $\frac{s-a}{(s-a)^2+\omega^2}$ (c) $\frac{6}{(s+2)(s-4)}$

[1+2+2+2]

OR

State convolution theorem. Show with an example that $1 * f(t) \neq f(t)$. Use convolution theorem to solve the initial value problem $y'' + 4y = g(t),\ y(0) = 3, y'(0) = -1$.
[1+2+4]

Q3. State and prove Cauchy Residue theorem for complex integration.
Evaluate $\oint_C \left(\frac{ze^{\pi z}}{z^4-16} + ze^{\pi/z}\right) dz$, where C is the ellipse $9x^2 + y^2 = 9$. [3+4]

SECTION "D"
[6Q.× 4=24 marks]

Q4. Define analytic and harmonic function. Verify that $u = x^3 - 3xy$ is harmonic in the whole complex plane and find a corresponding conjugate harmonic function v of u.

Q5. State second shifting theorem for Laplace transform. Find the transform of the function

$$f(t) = \begin{cases} 2 & \text{if } 0 < t < \pi \\ 0 & \text{if } \pi < t < 2\pi \\ \sin t & \text{if } t > 2\pi \end{cases}$$

Q6. Solve the PDE $4u_{xx} - u_{yy} = 0$, using the transformation $\xi = x + 2y$ and $\eta = x - 2y$.

Q7. Derive the polar form of Laplace equation $u_{xx} + u_{yy} = 0$.

Q8. Show that $\frac{d}{dx}[x^{\nu}J_{\nu}(x)] = x^{\nu}J_{\nu-1}(x)$, where the symbols have their usual meanings.

Q9. Evaluate $\int_{-\infty}^{\infty}\frac{dx}{(1+x^2)^3}$.

SECTION "E"

[5Q.× 2=10 marks]

Q10. Use Power series method to solve the differential equation $y' = 2xy$.

Q11. Solve $x^2y'' - 3xy' + 4y = 0$.

Q12. Find the orthogonal trajectories to the curve $y = cx^{3/2}$.

Q13. Find the linear fractional transformation which maps the points $0, 1, \infty$ onto $-1, -i, 1$ respectively.

Q14. Find the principal value of $(1 + i)^{-1+i}$.

KATHMANDU UNIVERSITY
End Semester Examination [C]
January, 2018

Marks Scored:

Level : B.E./B. Sc.
Year : II

Course : MATH 207
Semester: II

Exam Roll No.:

Time: 30 mins

F. M. : 20

Registration No.:

Date : JAN 1 4 2018

SECTION "A"

[10Q.× 1=10 marks]

Fill in the blank space (s) by writing the most appropriate word(s) or symbol(s).

1. The order of the differential equation for $y = A \text{ Cos } x + B \text{ Sin } x$, by eliminating arbitrary constants, is_______________.

2. The roots of the characteristic equation of a certain linear homogenous differential equation are 0,-4,2 and 2±4i. The general solution of this differential equation is_______________.

3. A certain second order linear homogenous differential equation has a fundamental set of solution consisting of $\{e^x, e^{-x}\}$. The solution satisfying y(0)=0, y'(0)=4 has the value at $x = 2$ of $y(2) =$ _______________.

4. The principal value of i^i is_______________.

5. The convolution $e^t * t =$_______________.

6. The two dimensional Laplace equation has the form_______________.

7. The Legendre's polynomial $P_1(x)$ is _______________.

8. The radius of convergence of the power series $\sum \frac{(2n)!}{(n!)^2}(z - 3i)^n$ is _______________.

9. The fixed points of $w = \frac{z-1}{z+1}$ are _______________.

10. The image of the region $1 \leq |z| \leq 3/2, \pi/6 \leq \theta \leq \pi/3$ under the mapping $w = z^2$ is_______________

SECTION "B"

[10Q.× 1=10 marks]

Fill in the blank space(s), DO NOT TICK, by selecting the most appropriate answers from among the given ones.

11. The integrating factor for the differential equation $xy' + (x+2)y = x^3$ is_______________

[$e^{\frac{x^2}{2}} + e^{2x}$; $e^{\frac{x^2}{2}+2x}$; x^2e^{2x}; $x^2 + e^{2x}$]

12. If $y = e^{2x}$ is a solution to $y'' - 5y' + ky = 0$, then the value of k is ____________
[-6; 0; -1; 6]

13. $J_{10}(x) =$ ____________________, where $J_{10}(x)$ is the Bessel's polynomial of order 10.
[$10J_{10}(x)$; $-J_{10}(x)$; $J_{10}(x) + J_{-10}(x)$; $J_{-10}(x)$]

14. The Laplace transform of the Dirac delta function $\delta(t-a)$ is ____________
[$\frac{e^{-as}}{s}$; e^{-as}; $e^{(s-a)}$; $\frac{e^{-as}}{s^2}$]

15. The solution $u(x, y)$ of partial differential equation $u_{xx} - u = 0$ is ____________
[$f(y)e^{x} + g(x)e^{-x}$; $f(y)e^{x} + g(x)e^{-y}$; $f(y)e^{x} + g(y)e^{-x}$; $(f(y) + g(x))e^{x}$]

16. A mass of 2 kilograms is attached to a string. The spring constant is 40. The mass is started in motion from the equilibrium position with an initial velocity of 1.5m/sec in the downward direction. Assume that there is no air resistance. The Initial Value Problem governing the position, u(t) of the mass. Assume the position is measured in meters and time in seconds____________

[$2u'' + 40u' = 0, u(0) = 0, u'(0) = 1.5$; $2u'' + 40u = 0, u(0) = 0, u'(0) = 1.5$;
$2u'' + 40u' = 0, u(0) = 1.5, u'(0) = 0$; $2u'' - 40u = 0, u(0) = 0, u'(0) = 0$;]

17. $e^{-\pi i} =$ ____________.
[-1; 0; 1; i]

18. The residue of the function $f(z) = \frac{z^4 - 3z^2 + 6}{(z+i)^3}$ at the singular point is____________.
[$-18\pi i$; $18\pi i$; πi; 0]

19. $\int_{-i}^{i} \frac{dz}{z} =$ ____________.
[1; 0; πi; $2\pi i$]

20. The differential equation $u_{xx} + 4u_{xy} + 4u_{yy} = 0$ is ____________.
[elliptic; parabolic; hyperbolic; circular]