KATHMANDU UNIVERSITY
End Semester Examination
January/February 2024

Marks Scored:

Level : B.E./B.Sc.
Year : II

11 FEB 2024

Course : MATH 207
Semester : II

Exam Roll No. :

Time: 30 mins.

F. M. : 20

Registration No.:

Date :

SECTION "A"

[10Q. × 1 = 10 marks]

Fill in the blank space(s) by writing the most appropriate word(s) or symbol(s).

1. The Laplace inverse of $\frac{1}{s(s+\pi)}$ = ______________________.

2. The degree of the differential equation $\left[\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2\right]^{1/2} = \left(\frac{d^3y}{dx^3}\right)^2$ is ______________.

3. The Wronskian of the functions $y_1 = 6^x$ and $y_2 = 6^{x+2}$ is ________________.

4. The type of the partial differential equation $u_{xx} - 6\,u_{xy} + 9\,u_{yy} = 0$ is ____________.

5. The Rodrigues' formula for Legendre's polynomial $P_n(x) = \alpha \frac{d^n}{dx^n}(x^2 - 1)^n$ where $\alpha =$ ________________.

6. $\int_C \frac{z}{z-1}\,dz =$ ________________ when C is $|z| = 2$ (clockwise).

7. The imaginary part of $\sin z$ is ________________________.

8. The complex function $e^{z/(z-2)}$ has an essential singularity at $z =$ ________________.

9. The general solution of the differential equation $y'' - 4y' + 3y = 0$ is $y(x) =$ ____________________________________, where a and b are constants.

10. If $f(z)$ is analytic in a simply connected domain D, and C is a simple closed curve that lies entirely within D, then $\int_C f(z)dz =$ __________________________.

SECTION "B"

[10 Q. × 1 = 10 Marks]

Fill in the blank space(s), **DO NOT TICK**, by choosing the most appropriate answers from among the given ones.

11. If $\mathcal{L}(f(t)) = F(s)$, then $\mathcal{L}(e^{2t} f(t)) =$ ____________________________.
[$e^{2s}F(x)$; $e^{-2s}F(s)$; $F(s-2)$; $F(s+2)$]

12. The function $y =$ ______________________ is a solution of the differential equation $y' + y \tan x = 0$, where c is a constant.
[$c \sin x$; $c \cos x$; $c \sin 2x$; $c \cos 2x$]

13. The equation $y' + f(x)y = 0$ has the general solution $y =$ ____________________, where c is a constant.
[$c \exp\left(\int f(x)dx\right)$; $c \exp\left(\int dx\right)$; $c \exp\left(-\int f(x)dx\right)$; $c \exp\left(-\int dx\right)$]

14. Given $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$, $0 < x < L$ subjected to initial conditions $u(x, 0) = f(x)$, and $\frac{\partial}{\partial t} u(x,t)\Big|_{t=0} = 0$, then the solution $u(x,t) =$ ______________________________.
[$f(x+t) + f(x-t)$; $f(x+t) - f(x-t)$; $\frac{1}{2}[f(x+t) + f(x-t)]$; $\frac{1}{2}[f(x+t) - f(x-t)]$]

15. The Bessel function $J_{-\frac{1}{2}}(x) =$ ____________________________.
[$\sqrt{\frac{2}{\pi x}} \sin x$; $\sqrt{\frac{2}{x}} \sin x$; $\sqrt{\frac{2}{\pi x}} \cos x$; $\sqrt{\frac{2}{\pi}} \cos x$]

16. The coefficient of z^2 in the expansion of $\frac{z}{z^2-1}$ when $|z| < 1$ is __________________.
[0; 1; 2; 3]

17. The complex function $f(z) = \bar{z}$ is analytic ______________________________.
[nowhere; only at $z = 0$; everywhere; only at $z = 1$]

18. $\lim_{z \to 0} \frac{f(z)}{z} =$ ____________________ when $f(z)$ is the complex conjugate of z.
[0; 1; $1 - i$; does not exist]

19. A Bernoulli differential equation is ________________________________.
[$y' + y = e^x$; $x^2 y'' + xy' + y = x$; $y' + \frac{y}{x} = x\, y^2$; $(1 - x^2)y'' - 2xy' + 6y = 0$]

20. The general solution of $\frac{dy}{dx} = 2x\, e^{x^2 - y}$ is ________________________ where c is a constant.
[$e^{x^2-y} = c$; $e^{-y} + e^{x^2} = c$; $e^y = e^{x^2} + c$; $e^y = e^{x^2} + c$]

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Semester : II
F. M. : 55

SECTION "C"
[3 Q. × 7 = 21 marks]

1. Let $f(z) = u(x,y) + i\,v(x,y)$ be defined and continuous in some neighborhood of a point $z = x + iy$ and differentiable at z itself. Then, prove that, at that point, the first-order partial derivatives of u and v exist and satisfy the Cauchy Riemann equations. Also, show that $\cos z$ is analytic, and $\frac{d}{dz}\cos z = -\sin z$. [4+3]

OR

If $f(z)$ is analytic in a simply connected domain D, then prove that the integral of $f(z)$ is independent of the path in D. Also, evaluate $\int_C f(z)\,dz$ when C is a straight line from $1 - i$ to $2 + i$ and $f(z) = (2x + 1) + i\,y$. [4+3]

2. Let a and b be any two constants, then explain in details the various cases to find the general solution of the homogeneous differential equation $\frac{d^2y}{dx^2} + a\frac{dy}{dx} + by = 0$.
Further, solve the initial value problem
$y'' + y' + 0.25y = 0,\ y(0) = 3.0,\ y'(0) = -3.5.$ [4+3]

3. Suppose the Laplace transform of $f(t),\ t \geq 0$ exists. Then, prove that
$$\mathcal{L}(f'') = s^2\,\mathcal{L}(f) - s\,f(0) - f'(0)$$
Use this relation to find the Laplace transform of $f(t)$, when $f(t) = t\cos\omega t$. [4 + 3]

SECTION "D"
[6 Q. × 4 = 24 Marks]

4. Find the type, normal form, and solve the partial differential equation
$u_{xx} - 2\,u_{xy} + u_{yy} = 0.$

OR

Find the solution $u(x,y)$ of the partial differential equation
$u_{xx} + 5\,u_x + 6u = 12\,(x + e^x)$ using solvable as ordinary differential equation method.

5. The Lagrange polynomial $P_n(x)$ of degree n is given by
$$P_n(x) = \sum_{m=0}^{M} \frac{(-1)^m (2n - 2m)!}{2^n\,m!\,(n-m)!\,(n-2m)!}\,x^{n-2m}$$
where $M = \frac{n}{2}$ or $\frac{n-1}{2}$, whichever is an integer. Use this relation to find the function $P_4(x)$.

6. Apply the Cauchy residue theorem to evaluate

$$\int_C \frac{e^z}{z^2(z^2+9)}\,dz$$

where C is a circle $|z - 1.5\,i| = 2$ (Counterclockwise).

7. Derive the model equation for an RLC circuit, and use it to find the current at time t in an RLC circuit given that $R = 4$ ohms, $L = 0.1$ henrys, $C = 0.05$ farads, and $E(t) = 100$ volts.

8. Solve the differential equation $\frac{dy}{dx} + 6x^2y = \frac{e^{-2x^3}}{x^2}$, $y(1) = 0$.

9. Solve the Euler-Cauchy equation $x^2y'' - x\,y' - 3y = 2x^3$ using the variation of parameters method.

SECTION "E"

[5 Q. × 2 = 10 Marks]

10. Find the Laplace transform of $\cos^2 at$, where a is a constant.

11. Find the radius of convergence of the power series $\sum_{n=0}^{\infty} 4^n(z+1)^n$.

12. Solve the differential equation $y' + y = 0$ using the power series method.

13. Find the solution $u(x,y)$ of the partial differential equation $u_{xy} = 0$ subjected to $u(x,0) = \sin x$ and $u(0,y) = e^y$.

14. Find an integrating factor, and solve $2xy\,dx + 3x^2dy = 0$.