KATHMANDU UNIVERSITY
End Semester Examination
February, 2025

Marks Scored:	

Level : B.E./B.Sc. Course : MATH 207
Year : II Semester : II
Exam Roll No. : Time: 30 mins. F. M. : 20
Registration No.: Date : 25 FEB 2025

SECTION "A"
[10Q. × 1 = 10 marks]

Fill in the blank space(s) by the most appropriate word(s) or symbol(s).

1. The degree of the ordinary differential equation $x\frac{d^2y}{dx^2} + y^4 = \left(\frac{dy}{dx}\right)^3$ is _____.

2. A curve that intersects each member of a given family of curves at right angles is called a/an _____ of the family.

3. Wronskian of the two functions $y_1 = \cos \omega x$, and $y_2 = \sin \omega x$ is _____.

4. If $f(z) = u(x, y) + i\, v(x, y)$ is analytic in a domain D, then it satisfies the Cauchy-Riemann equations _____ and _____.

5. The inverse Laplace transform of the function $F(s) = \frac{12}{(s+1)^3}$ is _____.

6. The one dimensional wave equation is given by _____.

7. The Legendre polynomial of degree 2, $P_2(x) =$ _____.

8. The residue of the function $f(z) = \frac{\sin z}{z^3}$ is _____ at $z = 0$.

9. $\oint_C e^z dz =$ _____ for any closed path C.

10. The mapping $f(z) = z^2 - 2z + 1$ is not conformal at $z =$ _____.

SECTION "B"

[10 Q. × 1 = 10 marks]

Fill in the blank space(s) by selecting the most appropriate answer from among the given ones. ***(DO NOT TICK THE ANSWER)***

11. $y = e^{2x}$ is a solution of $y'' - 5y' + ky = 0$, if $k =$ _____.
[-6; -1; 0; 6]

12. The differential equation $(\ell x + my)dx + (px + qy)dy$ will be exact if and only if _____, where $\ell, m,\ p$ and q are constants.
[$\ell = m$; $m = p$; $\ell = q$; $m = q$]

13. While solving differential equation $2y'' + 3y' = 2e^x$ by the method of undetermined coefficients, the choice for the particular-solution y_p is_____ where K is a constant.
[e^{Kx}; Ke^x; Kxe^x; $x + Ke^x$]

14. For any integer n, $J_{-n}(x) =$ _____, where the symbols have their usual meanings.
[$-J_n(x)$; $J_n(x)$; $(-1)^n J_n(x)$; $J_n(-x)$]

15. The Laplace transform of the function $f(t) = t - 3\ (t \geq 3)$ is _____.
[$\frac{1}{(s-3)^2}$; $\frac{1}{s^2} - \frac{3}{s}$; $\frac{e^{-3s}}{s-3}$; $\frac{e^{-3s}}{s^2}$]

16. The convolution $(1 * t)$ is _____.
[$\frac{1}{t}$; $\frac{t}{2}$; $\frac{t^2}{2}$; t]

17. The heat equation $u_t = c^2 u_{xx}$ is_____.
[Elliptic; Hyperbolic ; Parabolic; Circular]

18. $\cos z =$ _____.
[$\cos x \cosh y + i \sin x \sinh y$; $\cos x \cosh y - i \sin x \sinh y$;
$\cos x \cosh y + \sin x \sinh y$; $\cos x \cosh y - \sin x \sinh y$]

19. The function $f(z) = ze^{1/z}$ has the singularity at $z = 0$, and this singularity is called _____.
[Essential Singularity; Pole;
Non-isolated Singularity; Removable Singularity]

20. The mapping $w = f(z) = \frac{1}{z}$ has fixed point(s), _____
[0; ∞ ; 0, 1; none]

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SECTION "C"
[3 Q. × 7 = 21 marks]

1. Define the order of a differential equation. Show that the integrating factor of the differential equation $y' + p(x)y = r(x)$ is $e^{\int p(x)dx}$. Solve the initial value problem $y' + y\tan x = \sin 2x, y(0) = 1$. [1+3+3]

2. Define unit step function. State and prove second shifting theorem of the Laplace transform. Find the inverse transform $f(t)$ of $F(s) = \frac{a(s+k)+b\pi}{(s+k)^2+\pi^2}$. Find the Laplace transform of $f(t) = e^{-t}\ (0 < t < \pi)$. [1+2+2+2]

OR

State and prove the Convolution theorem of the Laplace transform. Solve the differential equation $y'' - y = e^{2t},\ y(0) = 0, y'(0) = 1$ using the Laplace transform method. [4+3]

3. Define analytic function. Evaluate $\oint_C \frac{4z}{(z-2)^2(z+1)}dz$ where C is a circle $|z| = 3$ (Counterclockwise). Show that $\int_0^{2\pi} \frac{d\theta}{\sqrt{2}-\cos\theta} = 2\pi$ using the residue integration method. [1+3+3]

SECTION "D"
[6Q. × 4 = 24 Marks]

4. Prove that $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\sin x$, where $J_\nu(x) = x^\nu \sum_{m=0}^{\infty} \frac{(-1)^m x^{2m}}{2^{2m+\nu}\, m!\, \Gamma(\nu+m+1)}$.

5. Solve $y'' - 5y' - 4y = 4x$ using the method of the variation of parameters.

6. Reduce to the normal form, and then solve $u_{xx} - 4u_{yy}$=0.

OR

Transform the Laplace equation $u_{xx} + u_{yy} = 0$ into polar coordinates.

7. Verify that $u(x,y) = xy^3 - x^3y$ is harmonic and determine its harmonic conjugate $v(x,y)$.

8. Compute $\int_C \bar{z}\ dz$ where C is the path from 0 to $1 + i$ along the parabola $y = x^2$. Also, evaluate $\int_0^{1+i} z\, dz$.

P.T.O.

9. A thermometer, reading $5°C$, is brought into a room whose temperature is $22°C$. One minute later the thermometer reading is $12°C$. How long does it take until the reading is practically $22°C$, say, $21.9°C$?

SECTION "E"

[5Q. × 2 = 10 Marks]

10. Find the orthogonal trajectories of $y = cx$.

11. Solve the third-order ODE $y''' + 9y' = 0$.

12. Find general values of the complex number, $\sqrt{1+i}$.

13. Find the Maclaurin series of $f(z) = \frac{1}{1+z^2}$.

14. Determine the bilinear transformation that maps $z_1 = 0,\ z_2 = 1,\ z_3 = \infty$ onto $w_1 = \infty,$ $w_2 = 1,\ w_3 = 0$ respectively.