KATHMANDU UNIVERSITY
End Semester Examination
February/March, 2019

FEB 1 8 2019

Level : B. E./B.Tech.
Year : II
Time : 2 hrs. 30 mins.

Course : MATH 207
Semester : I
F. M. : 55

SECTION "C"
[3 Q. × 7 = 21 marks]

1. Define harmonic function. If a complex function $f(z) = u(x,y) + i\,v(x,y)$ be an analytic function, then prove that $u(x,y)$ and $v(x,y)$ are harmonic. Show that the function $v(x,y) = x^2 - y^2$ is harmonic, and hence find its conjugate harmonic function. [1 + 2 + 4]

OR

State and prove Cauchy Residue theorem. Use this theorem to evaluate the complex integral

$$\int_C \frac{z^2}{(z-1)^2(z-\pi)}\,dz$$

where $C: |z-1| = 3$(Counterclockwise). [1 + 2 + 4]

2. Classify the second order partial differential equation

$$A\,u_{xx} + 2B\,u_{xy} + C\,u_{yy} = f(x,y,u,u_x,u_y)$$

where A, B and C are functions of x and y or constants. Classify the one dimensional wave equation

$$u_{tt} = \alpha^2\,u_{xx}$$

where α^2 is a constant, and hence find its normal form, and solve it. [2 + 5]

3. Derive the characteristic equation of the second order homogeneous differential equation of the form

$$\frac{d^2y}{dx^2} + a\frac{dy}{dx} + by = 0$$

where a and b are constants, and hence discuss the different cases for general solution. Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 12\,y = 0$$

[2 + 3 + 2]

SECTION "D"
[6 Q. × 4 = 24 Marks]

4. State first shifting theorem for Laplace transform. Use this theorem to find the Laplace transform of $e^{\pi t}\sin 2t$.

OR

State Convolution theorem $f * g$ of two functions $f(t)$ and $g(t)$, and hence compute $e^t * e^{-t}$.

5. Find the radius of convergence of the power series

$$\sum_{n=0}^{\infty} \frac{(-1)^n z^{2n+1}}{(2n+1)!}$$

6. Solve the Euler Cauchy equation $x^2 \frac{d^2y}{dx^2} + 3x\frac{dy}{dx} + y = 3x^6$.

7. Evaluate the Complex integral $\oint_C \frac{\sin z^2}{(z-3)^2} dz, C: |z-1| = 1$ (Counterclockwise).

8. If $u(x,t) = e^{-\pi^2 t} \sin 4x$ is a solution of one dimensional heat equation $u_t = \alpha^2 u_{xx}$. Find suitable α.

9. Let $P_n(x)$ be a Legendre polynomial of degree n, and the Rodrigue formula is given by

$$P_n(x) = \frac{1}{2^n\, n!} \frac{d^n}{dx^n} (x^2-1)^n, \qquad n = 0, 1, 2, \cdots$$

Use this Rodrigue formula to find the Legendre polynomial $P_4(x)$.

SECTION "E"

[5 Q. × 2 = 10 Marks]

10. Prove that $\cos hiz = \cos z$.
11. Find the inverse Laplace of $\frac{1}{(s+1)^2}$.
12. Solve $\frac{dy}{dx} + 1 = e^{x+y}$.
13. Find the Power series solution of $\frac{dy}{dx} = y$.
14. Find the solution $u(x,y)$ of the partial differential equation $\frac{\partial u}{\partial x} = u$.

Marks Scored:

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Registration No.:

Date : FEB 18 2019

SECTION "A"
[10 Q. × 1 =10 marks]

Fill in the blank space(s) by the most appropriate answer(s):

1. The degree of the differential equation $x^2 \frac{\partial^2 u}{\partial x^2} + u \left(\frac{\partial u}{\partial t}\right)^2 = 0$ is ________________.

2. The Laplace inverse of $\frac{1}{(s-1)(s-2)} =$ ________________.

3. The solution of the equation $\frac{dy}{dx} = 1 + y^2,\ y(0) = 0$ is ________________.

4. The Principal value of argument of the complex number $(1+i)^{12} =$ ________________.

5. The partial differential equation $xu_{xx} + u_{yy} = 0$ is parabolic if ________________.

6. The value of the complex integral $\oint_C \sin z\, dz =$ ________________, when $C: |z-1| = 2$ (Clockwise).

7. If $P_n(x)$ be a Legendre polynomial of degree n, the $P_1(x) =$ ________________.

8. The Wronskian of two functions e^x and e^{2x} is ________________.

9. The complex function $f(z) = z^2$ is conformal everywhere in the complex plane except at the point(s) where $z =$ ________________.

10. The orthogonal trajectories of the curve $xy = c$ is ________________, where c is a constant.

SECTION "B"
[10 Q. × 1 = 10 Marks]

Fill in the blank space(s), **DO NOT TICK**, by choosing the most appropriate answers from among the given ones.

11. If $L\{f(t)\} = F(s)$, then $L\{e^{-at} f(t)\} =$ ________________.

    [$\frac{F(s)}{s}$; $\frac{F(-s)}{s}$; $F(s+a)$; $F(s-a)$]

12. The partial differential equation $u_t = c^2 u_{xx}$ is a one dimensional ________________ equation.

    [wave; heat; Laplace; Poisson]

13. The radius of convergence of the power series $\sum_{n=0}^{\infty} (-1)^n z^n$ is ________________.

    [$\frac{1}{2}$; 1; $\frac{3}{2}$; ∞]

14. If a differential equation of the form $M(x,y)dx + N(x,y)dy = 0$ is not exact, and $\frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = f(x)$, then the integrating factor of the equation is ______________.

[$e^{-\int f(x)dx}$; $e^{\int f(x)dx}$; $e^{-\int f(y)dy}$; $e^{-\int f(y)dx}$]

15. The principal value in the general power $(-1)^i$ is ______________.

[$e^{-\frac{\pi}{2}}$; $e^{\frac{\pi}{2}}$; $e^{-\pi}$; e^{π}]

16. If $f(t) = \begin{cases} 1, & 0 \leq t \leq 1 \\ 0, & t > 0 \end{cases}$. The $L\{f(t)\} =$ ______________.

[$\frac{e^{-s}}{s}$; $-\frac{e^{-s}}{s}$; $\frac{1-e^{-s}}{s}$; $\frac{e^{-s}-1}{s}$]

17. $\lim_{z \to 1+i} \left(\frac{z-1-i}{z^2-2z+2}\right)^2 =$ ______________.

[$-\frac{1}{4}$; $-\frac{1}{2i}$; $\frac{1}{2i}$; $\frac{1}{4}$]

18. The model equation of an RLC-electric circuit is $L\frac{d^2I}{dt^2} + R\frac{dI}{dt} + \frac{1}{C}I = \frac{dE}{dt}$ where $E(t)$ is the voltage at time t, then the circuit is said to be critically damped if ______________.

[$R^2 = \frac{4L}{C}$; $R^2 - \frac{4L}{C} > 0$; $R^2 - \frac{4L}{C} < 0$; $R^2 = 4LC$]

19. The Newton's law of heating and cooling model is $\frac{dT}{dt} = \alpha\,(T - T_\infty)$ where $\alpha < 0, T$ and T_∞ are the temperature of the object and the surrounding medium, respectively. Then the ultimate temperature of the object is ______________.

[0; T; T_∞; $T - T_\infty$]

20. $\oint_C \frac{dz}{z-1} =$ ______________ when $C: |z-1| = 2$ (Clockwise)

[0; $2\pi i$; $-2\pi i$; $2\pi i e$]