Marks Scored:

KATHMANDU UNIVERSITY
End Semester Examination
February/March, 2018

Level: B. E./B. Sc./B. Tech. Course : MATH 207
Year : II Semester : I

Exam Roll No. : Time: 30 mins. F. M. : 20

Registration No.: Date FEB 2 7 2018

SECTION "A"
[10 Q. × 1 =10 marks]

Fill in the blank space(s) by writing most appropriate word(s) or symbol(s):

1. The principal value of argument of the complex number $(1+i)^2$ is ________________.

2. The Laplace transform of the function $f(t)=\begin{cases}1, & 0<t<a\\ 0, & t>a\end{cases}$ is ________________.

3. Taylor series of $f(x)$ about center at zero is known as ________________ series.

4. The order of a Laplace equation $u_{xx}+u_{yy}=0$ is ________________.

5. The Wronskian of x and $|x|$ is ________________.

6. The particular solution $y(x)$ of the differential equation $y''-y=0,\ y(0)=1,\ y'(0)=0$ is ________________.

7. If b be the residue of the complex function $f(z)$ at z_0 and C be a positively oriented simple closed curve containing only one isolated singular point z_0, then $\oint_C f(z)dz=$ ________________.

8. The Laplace inverse of $\frac{1}{(s-1)^2}$ is ________________.

9. $\oint_C e^{z+1}dz=$ ________________, $C:|z|=1$ (Counterclockwise).

10. The function $y=ce^{2x}$ is a solution of the differential equation ________________, where c is an arbitrary constant.

SECTION "B"
[10 Q. × 1 = 10 marks]

Fill in the blank space(s) by selecting the most appropriate answer from among the given ones:

11. $\lim_{z\to 0}\frac{\bar{z}}{z}=$ ________________.

[0; 1; ∞; does not exist]

12. Suppose $f(t)$ is a piecewise continuous function defined for $t \geq 0$, and $L\{f(t)\} = F(s)$. Then $L\{f(t-a)u(t-a)\} =$ ________________, where $u(t-a)$ is a unit step function.

[$e^{as}F(s)$; $e^{-as}F(s)$; $\frac{F(s)}{e^{as}}$; $\frac{F(s)}{e^{-as}}$]

13. If $J_n(x)$ be a Bessel polynomial of order n, and n is a positive integer, then $J_{21}(x) =$ ________ $J_{-21}(x)$.

[$-\frac{1}{2}$; -1; $\frac{1}{2}$; 1]

14. A one dimensional heat equation $u_t = \alpha u_{xx}$, $\alpha > 0$ is ________________.

[parabolic; hyperbolic; elliptic; mixed type]

15. The degree of the differential equation $\rho^2\left(\frac{d^2y}{dx^2}\right)^2 = \left[1+\left(\frac{dy}{dx}\right)^3\right]^{1/2}$ is ________________.

[1; 2; 3; 4]

16. The mass spring balance model without damping force, and external force is given by ________________, where m the mass of a block is attached in the spring, k is the spring constant and $x(t)$ is the displacement of the block from its equilibrium position.

[$mx''+kx'=0$; $mx''-kx'=0$; $mx''+kx=0$; $mx''-kx=0$]

17. The singular point $z=0$ of the function $f(z)=\cos\left(\frac{1}{z}\right)$ is of the type ________________.

[regular; removable; essential; simple pole]

18. The solution $u(x,y)$ of the partial differential equation $u_y = 0$ is ________________.

[x; y; function of x; function of y]

19. The Laplace transform of $t^{1/2}$ is ________________.

[$\frac{1}{s^{3/2}}$; $\frac{1}{2s^{3/2}}$; $\frac{\sqrt{\pi}}{s^{3/2}}$; $\frac{\sqrt{\pi}}{2s^{3/2}}$]

20. The orthogonal trajectories of the curve $y = cx^2$, where c is a constant, gives a family of ________________.

[parabolas; hyperbolas; ellipses; circles]

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Semester : I
F. M. : 55

SECTION "C"
[3 Q. × 7 = 21 marks]

1. State the Cauchy integral theorem. Use this theorem to prove that if $f(z)$ is analytic in a simply connected domain D, then the integral of $f(z)$ is independent of path in D. Evaluate the contour integral $\oint_C \frac{\sin z^2+e^z}{z^2-16}\, dz$, where $C: |z| = 3$ (Counterclockwise). [2 + 3 + 2]

OR

If $f(z) = u(x, y) + iv(x, y)$ is analytic in a domain D, then prove that the first order partial derivatives of u and v exist, and satisfy the Cauchy-Riemann equations $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ and $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$.
Also show that the function $f(z) = \sin z$ satisfies the Cauchy-Riemann equations. [5 + 2]

2. Define the classification of a second order partial differential equation

$$A(x, y)u_{xx} + 2B(x, y)u_{xy} + Cu_{yy} = f(x, y, u, u_x, u_y)$$

Classify the partial differential equation $u_{xx} + 4u_{xy} + 4u_{yy} = 0$, and solve it using d'Alembert's method. [2 + 5]

3. Derive the characteristic equation of a second order homogeneous differential equation

$$y''(x) + a\, y'(x) + b\, y(x) = 0$$

where a and b are constants. If the characteristic equation of this second order homogeneous differential equation has real and equal roots, say λ, then show that the general solution of this differential equation is of the form $y(x) = (c_1 + xc_2)e^{\lambda x}$ where c_1 and c_2 are arbitrary constants.
Find the general solution of the second order homogeneous differential equation

$$4y''(x) - 20y'(x) + 25y(x) = 0.$$

[2 + 3 + 2]

SECTION "D"
[6 Q. × 4 = 24 marks]

4. Evaluate, $\oint_C \frac{z^2+1}{z^2-z}\, dz$, $C: \left|z - \frac{1}{2}\right| = 1$ (Clockwise)

OR

Evaluate $\oint_C \operatorname{Re} z^2\, dz$, $C: |z| = 1$ (Counterclockwise)

5. Show that $J_{-1/2}(x) = \sqrt{\frac{2}{\pi x}} \cos x$, where the symbol has its usual meaning.

6. Use Laplace transform method to solve the initial value problem

$$y''(t) + 3y'(t) - 4y(t) = e^t,\ y(0) = 2,\ y'(0) = 0.$$

7. If u is independent of θ, show that the polar form of $u_{xx} + u_{yy}$ is $u_{rr} + \frac{1}{r} u_r$.

8. A thermometer, reading 10^0 C, is brought into a room whose temperature is 23^0 C. Two minutes later the thermometer reading is 18^0 C. How long will it take until the reading is 22^0 ?

9. Solve the non-homogeneous Euler-Cauchy equation $x^2 y''(x) - 2xy'(x) + 2y(x) = x^3 \sin x$.

<u>SECTION "E"</u>

[5 Q. × 2 = 10 marks]

10. Use first shifting theorem to find the Laplace inverse of $\frac{s-a}{(s-a)^2 + 4}$.

11. Find the solution $u(x,y)$ of the partial differential equation $u_x = u$ treating like an ordinary differential equation.

12. If $P_n(x)$ is a Legendre polynomial of degree n, then show that $P_2(x) = \frac{1}{2}(3x^2 - 1)$.

13. Solve the differential equation $(x + y + 1)\frac{dy}{dx} = 1$.

14. Show that the function $f(z) = z^2$ is analytic.