KATHMANDU UNIVERSITY
End Semester Examination [C]
December, 2024

Marks Scored:

Level : B.E./B.Sc./B.Tech.
Year : II

Course : MATH 207
Semester : I

Exam Roll No. :
Time: 30 mins.
F. M. : 20

Registration No.:
Date : 24 DEC 2024

SECTION "A"
[10 Q. × 1 = 10 marks]

Fill in the blank space(s) by most appropriate words or symbol(s):

1. The equation $y\left[1+\left(\frac{dy}{dx}\right)^2=c\right]$, where c is a constant is ____________________ ________________ differential equation.

2. The differential equation of the form $x^2y''+xy'+(x^2-v^2)y=0$ is called________ ________________________.

3. A differential equation of the form $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=f(x,y)$ is ____________________ equation.

4. The convolution of $e^t * e^{-t}$ is ______________________________.

5. The inverse Laplace transform $L^{-1}\{e^{-5s}\}=$ ____________________________.

6. The second order partial equation $u_{xx}+2u_{xy}+u_{yy}=0$ is ____________________.

7. If $f(z)=u(x,y)+iv(x,y)$ is analytic function in a domain D, then u and v satisfy Cauchy-Riemann equations __________________ and ____________________.

8. The number of isolated singular points of $f(z)=\frac{z-2}{(z+1)(z^2+1)}$ is ________________.

9. The trigonometric function $cosiz$ is defined by ____________________________.

10. The Laurent function $f(z)=z^2e^{1/z}$ ______________________ at $z=0$.

SECTION "B"
[10 Q. × 1 = 10 marks]

*Fill in the blank space(s), **DO NOT TICK,** by selecting the most appropriate answers from among the given ones.*

11. The Cauchy principal value of $\int_{-\infty}^{\infty}\frac{dx}{x^2-1}$ is ---------------------------------------.

    [0; $-\pi i$; $\frac{\pi}{2}i$; πi]

12. If w denotes the wornskian then $w(e^x, xe^x) =$ ______________________.
[e^{2x}; xe^{2x}; $2xe^{2x}$; e^{x^2}]

13. The Laplace transform of $f(t) = t^n e^{at}$ is ______________________.
[$\frac{n!}{(s-a)^n}$; $\frac{n!}{s^n}$; $\frac{n!}{(s-a)^{n+1}}$; $\frac{(n-1)!}{s^n}$]

14. The necessary condition for $M(x,y)dx + N(x,y)dy = 0$ to be an exact is ______________________.
[$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$; $\frac{\partial M}{\partial x} = \frac{\partial N}{\partial y}$; $\frac{\partial M}{\partial x} = \frac{\partial N}{\partial x}$; $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial y}$]

15. If z is a complex variable, then $|sinz|^2 =$ ______________________.
[$sin^2x + \sinh^2 y$; $sin^2x - \sinh^2 y$; $cos^2x - \sinh^2 y$; $cos^2x - i\sinh^2 y$]

16. The function $f(z) = z^3 + \frac{2}{z} - \frac{3}{z^2}$ has ______________________ at $z = 0$.
[pole of order 2; pole of order 1; pole of order 3; essential singularity]

17. $L\{u(t+a)\} =$ ______________________.
[$\frac{e^{as}}{s}$; e^{-as}; e^{-at}; $\frac{e^{-at}}{t}$]

18. The residue of $f(z) = \frac{cosz}{z}$ at $z = 0$ is ______________________.
[1; -1; $\frac{1}{2}$; $-\frac{1}{2}$]

19. The principal value of $\ln(-i) =$ ______________________.
[πi; $-2\pi i$ $\pi i/2$; $i3\pi/2$]

20. The Taylor's series $\sum_{n=0}^{\infty} z^n$ for all $|z| < 1$ converges to the function $f(z) =$ ______________________.
[$\frac{1}{z+1}$; $\frac{1}{1-z}$; $\frac{1}{z}$; e^z]

KATHMANDU UNIVERSITY
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Level : B.E./B.Sc./B.Tech.
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24 DEC 2024

Course : MATH 207
Semester : I
F. M. : 55

SECTION "C"
[3 Q. × 7 = 21 marks]

1. Define basis of solutions of second order homogeneous linear differential equation $y'' + p(x)y' + q(x)y = 0$. If one solution y_1 of $y'' + p(x)y' + q(x)y = 0$ is known then how to obtain other basis of solution? Find the general solution of $y'' + 10y' + 25y = e^{-5x}$ using method of undetermined coefficients. [1+3+3]

2. State and prove Convolution Theorem and use it to find the inverse Laplace transform of $F(S) = \frac{w}{s^2(s^2+w^2)}$. [4+3]

3. Write Cauchy-Riemann equations of an analytic function. Prove that if $f(z) = u(x,y) + iv(x,y)$ is analytic in a domain D then $\nabla^2 u = \nabla^2 v = 0$. Find the harmonic conjugate of $u = ln|z|$ and corresponding analytic function $f(z) = u + iv$. [1+2+3+1]

OR

Define residue of a complex function $f(z)$. Describe the method of calculating residue at simple poles and pole of m^{th} order. Calculate using residue integration method $\oint_c \frac{z^2-6}{2z-i} dz,\ c: |z| = 1$, a unit circle taken counterclockwise. [1+2+4]

SECTION "D"
[6 Q. × 4 = 24 marks]

4. Define Integrating factor. Solve the initial value problem $2sin(y^2)dx + xycos(y^2)dy = 0, y(2) = \sqrt{\pi}$.

5. Does a partial differential equation differ from an ordinary differential equation? Find the solution of $u_{xx} - 2u_{xy} + u_{yy} = 0$ where the symbols have their usual meaning.

6. Prove that $J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\, cosx$, where $J_n(x)$ is the Bessel's polynomial of degree n.

P.T.O.

7. Use Euler Cauchy form of Differential Equation of second order and find the general solution of $(x^2D^2 - 3xD + 4)y = 0$.

OR

Use Bessel's function $J_n(x)$ of the first kind of order n to evaluate $J_{\frac{3}{2}}(x)$.

8. Evaluate: $\oint_C \frac{dz}{z^2+4} dz$, C is the ellipse $4x^2 + (y-2)^2 = 4$, using Cauchy integral formula.m

9. Find the radius of convergence and center of the circle of convergence of the power series $\sum_{n=0}^{\infty} \frac{(3n)!}{2^n(n!)^3} z^n$.

SECTION "E"
[5 Q. × 2 = 10 marks]

10. Find the orthogonal trajectories of $y = ce^{-x}$..

11. Show that Legendre polynomial of degree 3 is $P_3(x) = \frac{1}{2}(5x^3 - 3x)$ where, $P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n}[(x^2-1)^n]$.

12. Find the principal value of $(i)^i$.

13. Find a bilinear transformation that maps $-1,\ 0, 1$ onto $1, -1\ \infty$.

14. Use power series method to find the solution $y' - y = 0$.