KATHMANDU UNIVERSITY
End Semester Examination [C]
November/December, 2023

Marks Scored:

Level : B.E./B.Sc./B.Tech.
Year : II

Course : MATH 207
Semester : I & II

Exam Roll No. :

Time: 30 mins.

F. M. : 20

Registration No.:

Date 03 DEC 2023

SECTION "A"

[10Q. × 1 = 10 marks]

Fill in the blank space(s) by writing most appropriate word(s) or symbol(s).

1. The solution of the differential equation $x\frac{dy}{dx} + y = 0$ is _____

2. The particular solution of the non homogeneous equation $y'' + p(x)y' + q(x)y = r(x)$ on an interval I using method of variation of parameters is $y_p(x) =$ _____ _____, where y_1 and y_2 are two linearly independent solutions of $y'' + p(x)y' + q(x)y = 0$, and w is the Wronskian of y_1 and y_2 .

3. If $J_\nu(x)$ is Bessel function of first kind of order ν then $\frac{d}{dx}[x^\nu J_\nu(x)] =$_____.

4. For integer $\nu = n$ the Bessel functions $J_n(x)$ and $J_{-n}(x)$ are _____ because $J_{-n}(x) = (-1)^n J_n(x)$.

5. The second order homogeneous linear ODE $y'' + ay' + by = 0$, where a and b are constants has two real roots if _____.

6. Laplace transform of the function $f(t) = (1 - e^{-at})\sin at$, where a constant is _____.

7. $f(z) = \frac{1}{z(z-1)}$ has a singular point(s) _____.

8. If R be the radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{2n}{n!}(z+i)^{2n-1}$, then R = _____.

9. If $w = \frac{az+b}{cz+d}$ $(ad - bc \neq 0)$ be a linear fractional transformation then the point at infinity(∞) is the image of the point z = _____.

10. If $f(z)$ is not analytic at z_o, then the coefficient b_1 in Laurent series expansion of $f(z)$ about z_o is called _____ of $f(z)$ at $z = z_o$.

SECTION "B"

[10 Q. × 1 = 10 marks]

Fill in the blank space(s), DO NOT TICK, by choosing the most appropriate answer from among the given ones.

11. $\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^4 + y = \sin(x)$ is a differential equation with degree _____ and order _____.

[3, 2; 2, 4; 1, 4; 2, 3]

12. For what value of k, $(x^3 + 3xy^2)dx + (kx^2y + y^3dx) = 0$, be an exact differential equation _____.

[3; 6; 2; x]

13. An electrical (or dynamical) system is said to be in steady state when the variables describing its behavior are periodic functions of time or _____.

[function of x; function of y; function of x and y; constants]

14. If the characteristics equation of the differential equation $y'' + ay' + by = 0$ has a real and repeated roots α. Then _____ is one of the solution of the differential equation.

[xe^{ax}; x^2e^{ax}; x^a; $x^a lnx$]

15. $L^{-1}\{1\} =$ _____, where L is a Laplace operator.

[1; $\delta(t)$; $u(t)$; dosen't exist]

16. The solution of PDE $u_x - u_y = 0$ is _____.

[$ce^{k(x+y)}$; $ce^{k(x-y)}$; $ce^{k(y-x)}$; $ce^{k(-x-y)}$]

17. If z is a complex variable then $|\cos z|^2 =$ _____.

[$\cos^2 x + \sinh^2 y$; $\cos^2 x + \cosh^2 y$ $\sin^2 x + \cosh^2 y$; $\sin^2 x + \sinh^2 y$]

18. $e^{-\pi i/2} =$_____.

[1; i; -1; -i]

19. The function $f(z) = 2z^{-3} - z^{-2} + 5z^{-1}$ _____ at infinity.

[is analytic; has pole of order 1; has pole of order 2; has pole of order 3]

20. The residue of $f(z) = z^{-4} \sin z$ at $z = 0$ is _____.

[$\frac{1}{6}$; $-\frac{1}{6}$; $\frac{1}{24}$; $-\frac{1}{24}$]

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Level : B.E./B.Sc./B.Tech.
Year : II
Time : 2 hrs. 30 mins.

Course : MATH 207
Semester : I & II
F. M. : 55

SECTION "C"
[3Q. × 7 = 21 marks]

1. Find the characteristic equation of the Euler-Cauchy equation $x^2y'' + axy' + by = 0$, where a and b are constants and then discuss the cases for general solution. Find the general solution of $(x^2D^2 + 7xD + 9)y = 0$, where D has its usual meaning. [4+3]

2. How does a partial differential equation differ from an ordinary differential equation? Show that the partial differential equation $u_{xx} - 4u_{xy} + 3u_{yy} = 0$ is hyperbolic and solve it by using the transformation $v = x + y$ and $z = 3x + y$. [1+1+5]

3. Define the harmonic and conjugate harmonic functions with example. Show that the function $v = -e^{-x}\sin y$ is harmonic. Also find its conjugate harmonic function u and analytic function $f(z) = u(x,y) + iv(x,y)$. [1+1+1+3+1]

OR

Define residue of a complex function $f(z)$. Describe the method of calculating residue at simple pole and pole of m^{th} order. Evaluate $\oint_c \frac{e^z}{\cos z}dz$, $c: |z| = 3$ (counterclockwise) using residue integration method. [1+3+3]

SECTION "D"
[6 Q. × 4 = 24 marks]

4. If $J_\nu(x)$ is the Bessel's function of first kind of order ν, then establish the relation $J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\cos x$.

5. Let $f(t)$ is continuous for all $t \geq 0$, it's Laplace transform exists and has a derivative $f'(t)$ that is piecewise continuous on every finite interval in the range $t \geq 0$. Then prove that $L(f') = sL(f) - f(0)$ and use it to find the Laplace transform of $f(t) = t\sin wt$.

OR

State convolution theorem and use it to find the inverse Laplace transform of $f(s) = \frac{s}{(s^2+\pi^2)^2}$.

6. Find two linearly independent solutions of the second order differential equation $y'' + 4y' + 4y = 0$ and hence its general solution. Find their Wronskian and use it to verify their linearly independence.

7. Find the principle value of $\int_{-\infty}^{\infty} \frac{dx}{(x^2-3x+2)(x^2+1)}$, only for the poles lying on the upper half of the plane.

8. Evaluate $\int_c \bar{z}\, dz$, where c is from $(1+i)$ to $3+i$ vertically and then from $3+i$ to $3+5i$ horizontally.

9. Express $f(t) = \begin{cases} 0, & 0 < t < 3 \\ 20\cos \pi t, & 3 < t < 6 \\ 0, & t > 6 \end{cases}$ into unit step function form and then find its Laplace transform.

SECTION "E"

[5 Q. × 2 = 10 marks]

10. Solve the differential equation $y' - y = e^{2x}$.

11. Experiments show that the rate of inversion of cane sugar in dilute solution is proportional to the concentration $y(t)$ of unaltered sugar. Let the concentration be $\frac{1}{100}$ at $t = 0$ and $\frac{1}{300}$ at $t = 4$ hours. Find $y(t)$.

12. Find the inverse Laplace transform of the function $\frac{1}{(s+1)(s+2)}$.

13. Solve the PDE: $u_{xy} - u_x = 0$.

14. Find the linear fractional transformation that maps the points $z_1 = -2, z_2 = 0, z_3 = 2$ onto the points $w_1 = \infty, w_2 = \frac{1}{2}, w_3 = \frac{3}{4}$ respectively.