KATHMANDU UNIVERSITY
End Semester Examination
August, 2025

Level : B.E./B.Sc.
Year : II
Time : 2 hrs. 30mins.

Course : MATH 207
Semester : II
F. M. : 55

SECTION "C"
[3 Q. × 7 = 21 marks]

1. Suppose that a complex function $f(z) = u(x,y) + i\, v(x,y)$ is analytic in a simply connected domain. Then, prove that the first-order partial derivatives of u and v exist and satisfy the Cauchy-Riemann equations $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$, and $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$.
Also, show that e^{2z} is analytic, and $\frac{d}{dz}(e^{2z}) = 2\, e^{2z}$. [4+3]

OR

State and Prove the Cauchy Residue Theorem. Use the Residue theorem to evaluate the complex integral $\int_C \frac{2z+3}{z^2-\frac{1}{4}}\, dz$, where $C: |z| = 1$ (Counterclockwise). [4+3]

2. Let $y_1(x)$ and $y_2(x)$ be two linearly independent solutions of the homogeneous differential equation
$\frac{d^2y}{dx^2} + a(x)\frac{dy}{dx} + b(x)y = 0$. Show that the particular integral $y_p(x)$ of the non-homogeneous differential equation $\frac{d^2y}{dx^2} + a(x)\frac{dy}{dx} + b(x)y = q(x)$ under the variation of parameters method is given by

$$y_p(x) = -y_1(x)\int \frac{y_2(x)\, q(x)}{W(x)}\, dx + y_2 \int \frac{y_1(x)\, q(x)}{W(x)}\, dy$$

where $W(x)$ is the Wronskian of $y_1(x)$ and $y_2(x)$. Also, find the particular integral $y_p(x)$ of the equation $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 5\, e^{-x}$ using the variation of parameters method. [4+3]

3. Suppose that the Laplace transform of $f(t),\ t \geq 0$ exists. Then, prove that

$$\mathcal{L}(f'') = s^2\, \mathcal{L}(f) - s\, f(0) - f'(0)$$

where $\mathcal{L}(f)$ represents the Laplace transform of $f(t)$. Use this relation to find the Laplace transform of $f(t)$, when $f(t) = \sin at$. [4+3]

SECTION "D"
[6Q. × 4 = 24 Marks]

4. Prove that for an integer n, Bessel functions $J_n(x)$ and $J_{-n}(x)$ are related by $J_{-n}(x) = (-1)^n J_n(x)$.

P.T.O.

5. Use D'Alembert's method to find the solution $u(x,t)$ to the partial differential equation $u_{tt} = c^2 u_{xx}$.

6. Evaluate the integral

$$\int_C \frac{z^4 - 3z^2 + 6}{(z+i)^3} dz$$

where C is a circle $|z - 0.5\, i| = 2.5$ (Counterclockwise).

OR

Use the complex integration method to evaluate the integral $\int_{-\infty}^{\infty} \frac{dx}{x^4 - 1}$.

7. Derive the differential equation model for an RLC circuit, and use it to find the current at time t in an RLC circuit given that $R = 4$ ohms, $L = 0.5$ henrys, $C = 0.1$ farads, and $E(t) = 500 \sin 2t$ volts.

8. Solve the linear first-order differential equation $\frac{dy}{dx} - \left(1 + \frac{3}{x}\right) y = x + 2, \;\; y(1) = e - 1$.

9. Use the Laplace transform method to solve the initial value problem

$y'' - y' - 2y = 2t, \;\; y(0) = 1, \;\; y'(0) = 0$, where $y' = \frac{dy}{dt}$ etc.

SECTION "E"

[5Q. × 2 = 10 Marks]

10. Find the solution $y(x)$ of the equation $\frac{dy}{dx} + 1 = e^{x+y}$.

11. Use the power series method to solve the ordinary differential equation $\frac{dy}{dx} - y = 0$.

12. Show that the function $u(x,y) = 2xy$ is harmonic.

13. Find an integrating factor, and solve $2xy\, dx + 3x^2 dy = 0$.

14. Expand the function $f(z) = \frac{1}{z^2 - 1}$ in a Laurent series that converges for $0 < |z - z_0| < R$ and determine the precise R when $z_0 = 1$.