Mark Scored :

KATHMANDU UNIVERSITY
End Semester Examination
August/September, 2017

Level : B. E./B. Sc.
Year : II

Course : MATH 207
Semester : II

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Exam Roll No. : Time: 30 mins. F. M. : 20

Registration No.: Date : SEP 1 4 2017

SECTION "A"

[10 Q. × 1 = 10 marks]

Fill in the blank space(s) by writing the most appropriate word(s) or symbol(s).

1. The order of the differential equation $\left(\frac{d^2y}{dx^2}\right)^{\frac{1}{2}} = \left(\frac{dy}{dx}\right)^2 + xy$ is ________________

2. The integrating factor of the linear differential equation $y' + \frac{y}{x} = x^2$ is ____________

3. If $y = x^2$ is a solution of the differential equation $x^2y'' + y = f(x)$, then $f(x) =$ ________________

4. The Legendre polynomial of degree n given by Rodrigues's formula is $P_n(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}[(x^2-1)^n]$, then $P_2(x) =$ ________________

5. The Laplace Transform of the cosine hyperbolic function $f(t) = cosh\, t$ is ________________

6. The inverse Laplace of $\frac{1}{s^3}$ is ____________

7. The heat equation $\frac{\partial u}{\partial t} - c^2\frac{\partial^2 u}{\partial x^2} = 0$ is ______________ type in PDE classification.

8. If $z = 3 - 2i$, then the product $z\bar{z}$ is __________, where $\bar{z}$ denotes the complex conjugate of z .

9. The value of the complex integral $\oint_C \frac{dz}{z} =$ ____________________, where $C: |z| = 1$ is counter-close wise.

10. The residue of $f(z) = \frac{2}{z^2+1}$ at the pole $z = i$ is ______________

SECTION "B"

[10 Q. × 1 = 10 marks]

Fill in the blank space(s), DO NOT TICK, by selecting the most appropriate answers from among the given ones.

11. The differential equation $y'' + p(x)y' + q(x)y = r(x)$ is said to be non-homogeneous if ________________

[$p(x) \equiv 0$; $q(x) \neq 0$; $r(x) \equiv 0$; $r(x) \neq 0$]

12. The differential equation $(Ax + By)dx + (Cx + Dy)dy$ will be exact if and only if ____________ where $A, B, C, and\ D$ are constants.

[$A = C$; $B = C$; $A = D$; $B = D$]

13. Two basis solution of the second order differential equation $y'' + 2y' + y = 0$ is ____________

[$\{e^x, e^{-x}\}$; $\{e^x, xe^{-x}\}$; $\{e^x, xe^x\}$; $\{e^{-x}, xe^{-x}\}$]

14. The Wronskian of $y_1 = 1, y_2 = x\ and\ y_3 = \frac{x^2}{2}$ is ____________

[1; x; x^2; x^3]

15. The radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{1}{\pi^n}\left(x - \frac{\pi}{2}\right)^n$ is ____________

[$\frac{1}{\pi}$; $\frac{2}{\pi}$; π; $\frac{\pi}{2}$]

16. Let $f(t)$ be a function with $f(0) = 0$ and the Laplace of its derivative $\mathcal{L}(f') = \frac{1}{s}$, then the Laplace of the function $f(t)$, $\mathcal{L}(f) =$ ____________

[1; t; e^t; $\sin t$]

17. The convolution $(1 * t) =$ ____________

[$\frac{1}{2}$; $\frac{t}{2}$; $\frac{t^2}{2}$; $\frac{e^t}{2}$]

18. The characteristics of the one dimensional wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ to reduce it into its normal form the equation $u_{vw} = 0$ are ....

[$v = x + t\ and\ w = x - t$; $v = x\ and\ w = x + ct$;
$v = x + ct\ and\ w = x - ct$; $v = x\ and\ w = t$]

19. The value of i^{19} =..................where i is imaginary unit.

[-1; i; $-i$; 1]

20. The function $f(z) = \frac{e^z}{z(z-2)^5} + \frac{\sin z}{(z-2i)^2}$ has a pole of order .... at $z = 2i$.

[1; 2; 5; 0]

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SEP 1 ? 2017

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Year : II
Time : 2 hrs. 30 mins.

Course : MATH 207
Semester : II
F. M. : 55

SECTION "C"
[3 Q. × 7 = 21 marks]

1. Derive the condition for the first order differential equation of the form $M(x, y)dx + N(x, y)dy = 0$ to become the exact form.

Solve the differential equation $(x^4 + y^2)dx - xydy = 0$ using the method of exactness by finding its appropriate integrating factor. Also, find the it's particular solution if $y(2) = 1$ is given. [3+4]

2. Explain the detail procedure to find the solution of the Euler-Cauchy second order differential equation of the form $x^2y'' + axy' + by = 0$, where a and b are real constants. Also, discuss three different possible solutions of this equation.
Solve the differential equation $x^2y'' - 4xy' + 6y = 0$ with given initial conditions $y(1) = 1, y'(1) = 0$. [1+3+3]

OR

Classify three different types of second order partial differential equation (PDE) of the form $Au_{xx} + 2Bu_{xy} + CU_{yy} = F(x, y, u, u_x, u_y)$, where the coefficients A, B, C are functions of x and y. Using D'Alembert's method, find the solution $u = u(x, t)$ of the one dimensional wave equation $u_{tt} = c^2u_{xx}$ with initial conditions $u(x, 0) = f(x)$ and $u_t(x, 0) = g(x)$. (Show the details of your work) [2+5]

3. Define a singular point and a pole of order m of a complex function $f(z)$. State and prove the residue theorem for the integration of the complex function $f(z)$.
Find the Cauchy principal value of the real integral $\int_{-\infty}^{\infty} \frac{dx}{x(x+4)(x^2+16)}$ using residue theorem. [1+2+4]

SECTION "D"
[6 Q. × 4= 24 marks]

4. Find the orthogonal trajectories of the family of curves $y = x + Ce^{-x}$.

5. Show that $\frac{d}{dx}(x^{-v}J_v(x)) = -x^{-v}J_{v+1}(x)$, where $J_v(x) = x^v \sum_{m=0}^{\infty} \frac{(-1)^m x^{2m}}{2^{2m+v} m!\, \Gamma(v+m+1)}$ is the Bessel's function of first kind of order v.

6. Define Laplace transform of a function $f(t), t \geq 0$. Show that the Laplace transform of t^α, $\mathcal{L}(t^\alpha) = \frac{\Gamma(\alpha+1)}{t^\alpha}$, where α is a real number and $\Gamma(.)$ denotes the gamma function.

OR

Using Laplace transform to solve the differential equation $y'' + 9y = 10e^{-t}$ with initial conditions $y(0) = 0$ and $y'(0) = 0$.

7. Show that the function $u = xy$ is harmonic and find its conjugate harmonic function v to get the corresponding analytic function $f(z) = u + iv$.

8. Develop the Laurent's series of the function $f(z) = \frac{2z}{z^2+4}$ at the point $z = 2i$. Also find the residue of $f(z)$ at the point $z = 2i$ from the series.

9. If an electromotive force of $160 \cos 5t$ $Volts$ is impressed on a series circuit composed of a 20 Ω resistor and 10^{-1} H inductor, find the steady state current in the circuit.

SECTION "E"

[5 Q. × 2 = 10 marks]

10. Find the power series solution of the first order differential equation $y' = -ky$.

11. Find the inverse Laplace of $\frac{6}{(s+2)(s-4)}$.

12. Find the principle value of the general power $(2i)^i$.

13. $\int_C \bar{z}\, dz$, where C is the shortest path from $1 + i$ to $3 + 2i$.

14. Find the linear fractional transformation that maps the points $i, -i, 0$ onto $0, \infty, -1$.