KATHMANDU UNIVERSITY
End Semester Examination [C]
April/May, 2023

Marks Scored:

Level : B.E./B.Tech.
Year : II

Course : MATH 207
Semester : I

Exam Roll No. :   Time: 30 mins.   F. M. : 20

Registration No.:   Date : 30 APR 2023

SECTION "A"

[10 Q. × 1 = 10 marks]

Fill in the blank space(s) by most appropriate word(s) or symbol(s).

1. The differential equation $Mdx + Ndy = 0$ is exact if ______________.

2. If the two roots of second-order homogenous differential equations are $2i$ and $-2i$, then the general solution $y =$ ______________.

3. If W denotes the Wronskian, then $W(e^x, xe^x) =$ ______________.

4. The value of $e^{\pi i}$ is______________.

5. The Laplace transform of t^2e^{3t} is______________.

6. The solution $u(x,y)$ of $u_x = u$ is ______________.

7. The Legendre's polynomial, $P_1(x)$ is ______________.

8. The radius of convergence of the power series $\sum \frac{n}{n+1} z^n$ is ______________.

9. The residue of $e^{1/z}$ has ______________________ singularity at $z = 0$.

10. The point where the mapping $f(z) = z^2 - 2z + 1$ is not conformal is______________.

SECTION "B"

[10 Q. × 1 = 10 marks]

Fill in the blank space(s), **DO NOT TICK**, by choosing the most appropriate answer from among the given ones.

11. While solving differential equation $y'' + 2y' + y = 3\sin x$ by the method of undetermined coefficients, the choice for the particular solution y_p is______________.
[$A\sin x$; $B\cos x$; $A\sin x + B\cos x$; $AB\sin x\cos x$]

12. Which of the following differential equation has degree 2 and order 2? ______________.

[$(y'')^2 + 2y' + y = 0$; $y'' + (y')^2 + y = 0$; $y'' + (y')^2 + y^2 = 0$; $\sqrt{y'' + y} = y'$]

13. The value of $J_{10}(x) =$ ______________, where $J_{10}(x)$ is the Bessel's function of order 10.

[$10J_{10}(x)$; $-J_{10}(x)$; $J_{10}(-x)$; $J_{-10}(x)$]

14. If $P_n(x)$ is a Legendre function of degree n, then $P_n(-x) =$ ______________.

[$-P_n(x)$; $(-1)^n P_n(x)$; $P_{-n}(x)$; $(-1)^n P_n(-x)$]

15. Convolution of t and 1, that is, $t * 1 =$______________

[1; t; $\frac{1}{2}t$; $\frac{1}{2}t^2$]

16. The inverse Laplace transforms of $\frac{1}{s(s-1)}$ is ______________.

[$1 - e^t$; $1 - e^{-t}$; $e^t - 1$; $e^{-t} - 1$]

17. Partial differential equation of the form $Au_{xx} + 2Bu_{xy} + Cu_{yy} = F(x, y, u, u_x, u_y)$ is classified as parabolic if ______________.

[$AC - B^2 < 0$; $AC - B^2 > 0$; $AC - B^2 = 0$; $AC - B^2 \neq 0$]

18. The function $f(z) = \frac{\sin z}{z^4}$ has pole of order ______________.

[1; 3; 4; 5]

19. The value of the integral $\oint_C \frac{e^{2z}}{z-1} dz$, $c: |z| = 2$ (counterclockwise) is ______________.

[0; e^2; $2\pi i\, e^2$; $2\pi i/e^2$;]

20. The mapping $w = f(z) = \frac{1}{z}$ has fixed point(s) ______________.

[0; ∞; $0, 1$; $-1, 1$]

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April/May, 2023

30 APR 2023

Level : B.E./B.Tech.
Year : II
Time : 2 hrs. 30 mins.

Course : MATH 207
Semester : I
F. M. : 55

SECTION "C"

[3 Q. × 7 = 21 marks]

1. If the differential $P(x,y)dx + Q(x,y)dy = 0$ is not exact, show that it has an integrating factor $e^{\int R(x)dx}$ where $R(x) = \frac{1}{Q}\left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right)$. Find the integrating factor and solve the differential equation $(x^3 + xy^4)dx + 2y^3dy = 0$. [4+3]

2. Define the Laplace transform of a function $f(t)$. State and prove the second shifting theorem of Laplace transforms. Find the Laplace transform of $\begin{cases} 0, & 0 < t < 1 \\ t^2, & 1 < t < 2. \\ 0, & t > 2 \end{cases}$ [1+3+3]

OR

State and prove the Convolution theorem of the Laplace transform. Solve the initial value problem $y'' + 2y' - 2y = 2t, y(0) = 1, y'(0) = 0$. [1+3+3]

3. If a function $f(z) = u(x,y) + iv(x,y)$ is analytic in a domain D, prove that the partial derivatives of u and v satisfy $u_x = v_y$ and $u_y = -v_x$ in D. Define the harmonic function and then show that $u(x,y) = xy^3 - x^3y$ is a harmonic function and find the conjugate harmonic function $v(x,y)$. [3+1+3]

SECTION "D"

[6Q. × 4 = 24 marks]

4. Solve $y'' - 2y' + y = \frac{12e^x}{x^3}$ by the method of variation of parameters.

5. Bessel function $J_\nu(x)$ is defined by $J_\nu(x) = x^\nu \sum_{m=0}^{\infty} \frac{(-1)^m x^{2m}}{2^{2m+\nu} m! \Gamma(\nu+m+1)}$. Prove that $J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \cos x$.

6. Reduce the wave equation $u_{tt} = c^2 u_{xx}$ into the normal form and solve it.

OR

Express the Laplace equation $u_{xx} + u_{yy} = 0$ in polar coordinates defined by $x = r\cos\theta$, $y = r\sin\theta$.

7. Evaluate the contour integral $\oint_c \frac{z^2}{(z-1)^2(z-2)} dz,\ c: |z| = 2.5$ (counterclockwise).

8. Show that $\int_0^{2\pi} \frac{dz}{\sqrt{2}-\cos\theta} = 2\pi$ using contour integral.

9. A thermometer, readings $5^\circ\ C$, is brought into a room whose temperature is $22^\circ C$. One minute later the thermometer reading is $12^\circ C$. How long does it take until the reading is practically $22^\circ\ C$, say, $21.9^\circ\ C$?

SECTION "E"

[5Q. × 2=10 marks]

10. Find the orthogonal trajectories of the family of curves $y = ke^x$, k is a constant.

11. Use the power series method to solve $y' - 2y = 0$.

12. Evaluate the Laplace inverse transform of $\frac{2s-56}{s^2-4s-12}$.

13. Find the general value and principal value of $(1-i)^i$.

14. Find the linear fractional transformation that maps $z_1 = 0, z_2 = 1,\ z_3 = \infty$ onto $w_1 = -1,\ w_2 = -i,\ w_3 = 1$ respectively.