

KATHMANDU UNIVERSITY
End Semester Examination
June/July, 2023

Marks Scored:

Level : B.Sc.
Year : II

Course : MATH 206
Semester : I

Exam Roll No.:

Time: 30 mins.

F. M. : 20

Registration No.:

Date : 30 JUN 2023

SECTION "A"
[10Q. \times 1 = 10 marks]

Fill in the blank space(s) by the most appropriate word(s) or symbol(s).

1. In analysis of variance, the _____ sum of squares measures the variability of the sample treatment means around the overall mean.
2. Rank the score of 5 in the following set of scores: _____
9, 3, 5, 10, 8, 5, 9, 7, 3, 4
3. If Pearson's correlation coefficient between stress level and workload is .8, how much is the R^2 or the variance in stress level is accounted for by workload or? _____
4. Non parametric test for comparing two groups _____
5. Non parametric test for comparing more than two groups _____
6. If all the points of a scatter diagram lie on a straight line falling from left upper corner to the right bottom corner, the correlation is called _____
7. Study of correlation among three or more variables simultaneously is called _____.
8. While computing F ratio, customarily, the larger variance is taken as _____.
9. Degrees of freedom for Chi-square test in case of contingency table of order (4 \times 3) is _____.
10. The control charts used for the number of defects per unit is _____.

SECTION "B"
[10Q. \times 1 = 10 marks]

Encircle the most appropriate answer among the given options.

11. In analysis of variance, the _____ sum of squares measures the variability of the observed values around their respective treatment means.
a. treatment b. error c. interaction d. total

12. If the true means of the k populations are equal, then Mean sum of squares due to treatment/Mean sum of square due to error should be:
 a. more than 1.00 b. close to 1.00 c. close to 0.00 d. close to -1.00
13. Which of the following sampling method is appropriate to study the prevalence of AIDS amongst male and female in Nepal in 2023?
 a. Cluster sampling b. Systematic sampling
 c. Quota sampling d. Stratified random sampling
14. If the coefficient of determination R^2 is equal to 1 in a regression problem, then _____.
 a. Error sum of squares must be 0 b. Total sum of squares must be 0
 c. Regression sum of squares must be 0 d. Residual sum of squares must be 1
15. For the chi-square goodness-of-fit-test, the calculated chi-square value is 7.21. If the tabulated chi-square value is 10.645, what is the appropriate decision for this test?
 a. Reject the null hypothesis
 b. Fail to reject the null hypothesis
 c. Accept both the null and the alternative hypothesis
 d. It is impossible to determine anything from the given information
16. Suppose that the estimated regression equation of a College of Business graduates is given by: $\hat{y} = 32,000 + 4,000x + 1,800D$, where y is the starting salary, x is the grade point average and D is a dummy variable which takes the value of 1 if the student is a finance major and 0 if not. An accountancy major graduate with a 3.5 grade point average would have a starting salary of
 a. Rs. 47,800 b. Rs. 46,000 c. Rs. 37,800 d. Rs. 32,000
17. Coefficient of determination equal to 0.78 means that _____.
 a. The model provides a poor fit
 b. Most of the variation in y is unexplained by the regression equation
 c. Model has problems
 d. Most of the variation in y is explained by the variation in the independent variables used in the model
18. Which one among the following variables cannot be expressed in quantitative terms?
 a. Numerical Aptitude b. Marital Status
 c. Socio-economic Status d. Professional Attitude
19. The magnitude of the difference between observed frequencies and expected frequencies is called _____.
 a. F value b. Z value c. t value d. Chi-square value
20. _____ is the simplest and most widely used non-parametric test
 a. Sign test b. Kruskal Wallis H test
 c. Chi-square test d. Mann Whitney U test

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Semester : I
F. M. : 55

SECTION "C"

[3Q. × 9 = 27 marks]

1. For the data given below fit a multiple linear regression model, $Y = \beta_0 + \beta_1x_1 + \beta_2x_2 + \epsilon$. Construct and interpret the line of regression. Compute R^2 . Also compute $r_{y \cdot x_1, x_2}$ and $R_{Y \cdot x_1, x_2}$ [3+1+2+1.5+1.5]

Recovery (y)	0.740	0.745	0.718	0.678	0.652
Flow Speed (X_1)	1	1.5	2	2.5	3
Type Furnace(X_2)	1	1	1	1	1

2. What is statistical process control? Target weight for a box of cereal is 350g. Each day a sample of 300 boxes is taken, and the number that are underweight, is counted. The number of underweight boxes for each of the last 24 days is as follows: [2+3.5+3.5]
- 23 12 19 19 20 19 21 27 26 23 22 25 30 30
22 25 27 29 35 39 43 41 39 29
- a. Compute upper and lower control limits for p chart and np chart.
b. Draw the control chart. Is the process in control? If not, when is it first detected to be out of control?

3. What are non-parametric tests? [2+4+3]
- a. A chemical engineer is studying newly developed polymer to be used in removing toxic wastes from water. The experiment was conducted in five different temperatures. The response noted is the percent of impurities removed by the treatment. Test the null hypothesis of equal treatment means at the a) 0.05 and b) 0.01 significance levels. Use Kruskal-Wallis H test
- Temperature I 40 35 42 48 50 51
Temperature II 36 42 38 39 37 40
Temperature III 49 51 53 53 52 50
Temperature IV 47 49 51 52 50 51

- b. The sequence of 48 tools produced by a machine shows that the following sequence of good (G) and defective (D) tools:
GGGGGGDDGGGGGGGGGGDDDDGGGGGGDGGGGGGGGGGDDGGGGGGDGG

SECTION "D"
[4Q. × 7 = 28 marks]

4. Two quantities x and y are measured and corresponding values are given in the following table.

x	20	40	60	80	100	120
y	5.5	9.1	14.9	22.8	33.3	46

Find a second-degree parabola Y on X to the data. Find the R^2 , [4+3]

5. A broth used to manufacture a pharmaceutical product has its sugar content, in mg/mL, measured several times on each of three successive days.

Day 1: 5.0 4.8 5.1 5.1 4.8 5.1 4.8 4.8 5.0 5.2 4.9 4.9 5.0

Day 2: 5.8 4.7 4.7 4.9 5.1 4.9 5.4 5.3 5.3 4.8 5.7 5.1 5.7

Can you conclude that the variability of the process is greater on the second day than on the first day.

6. A model for predicting the extraction of manganese in % (y) from particle size in mm (x_1), the amount of sulfur dioxide needed (x_2), and the duration of leaching in minutes (x_3) is given as

$$y = 56.145 - 9.046x_1 - 33.421x_2 + 0.243x_3 - 0.5963x_1x_2 - 0.0394x_1x_3 + 0.6022x_2x_3 + 0.6901x_1^2 + 11.7244x_2^2 - 0.0097x_3^2$$

There were a total of $n = 27$ observations, with $SSE = 209.55$ and $SST = 6777.5$

[3+2+2]

- Predict the extraction percent when the particle size is 3 mm, the amount of sulfur dioxide is 1.5 and the duration of leaching is 20 minutes.
 - Is it possible to predict the change in extraction process when the duration of leaching increases by one minute, keeping the effect of other variables constant?
 - Compute the coefficient of determination R^2
7. Four lighting methods were used in each of three rooms. The only effect of interest is the lighting type; the room is a blocking factor. The following sums of squares were calculated; sum of squares for blocks = 11432, sum of squares for treatments = 9943 and total sum of squares = 51376
- Construct an ANOVA table
 - Is there a difference in the lighting types?

Table II Cumulative Standard Normal Distribution

$$\Phi(z) = \int_{-\infty}^z \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt$$

z	0.00	0.01	0.02	0.03	0.04	z
0.0	0.50000	0.50399	0.50798	0.51197	0.51595	0.0
0.1	0.53983	0.54379	0.54776	0.55172	0.55568	0.1
0.2	0.57926	0.58317	0.58706	0.59095	0.59483	0.2
0.3	0.61791	0.62172	0.62551	0.62930	0.63307	0.3
0.4	0.65542	0.65910	0.66276	0.66640	0.67003	0.4
0.5	0.69146	0.69497	0.69847	0.70194	0.70540	0.5
0.6	0.72575	0.72907	0.73237	0.73565	0.73891	0.6
0.7	0.75804	0.76115	0.76424	0.76730	0.77035	0.7
0.8	0.78813	0.79103	0.79389	0.79673	0.79954	0.8
0.9	0.81594	0.81859	0.82121	0.82381	0.82639	0.9
1.0	0.84134	0.84375	0.84613	0.84849	0.85083	1.0
1.1	0.86433	0.86650	0.86864	0.87076	0.87285	1.1
1.2	0.88493	0.88686	0.88877	0.89065	0.89251	1.2
1.3	0.90320	0.90490	0.90658	0.90824	0.90988	1.3
1.4	0.91924	0.92073	0.92219	0.92364	0.92506	1.4
1.5	0.93319	0.93448	0.93574	0.93699	0.93822	1.5
1.6	0.94520	0.94630	0.94738	0.94845	0.94950	1.6
1.7	0.95543	0.95637	0.95728	0.95818	0.95907	1.7
1.8	0.96407	0.96485	0.96562	0.96637	0.96711	1.8
1.9	0.97128	0.97193	0.97257	0.97320	0.97381	1.9
2.0	0.97725	0.97778	0.97831	0.97882	0.97932	2.0
2.1	0.98214	0.98257	0.98300	0.98341	0.98382	2.1
2.2	0.98610	0.98645	0.98679	0.98713	0.98745	2.2
2.3	0.98928	0.98956	0.98983	0.99010	0.99036	2.3
2.4	0.99180	0.99202	0.99224	0.99245	0.99266	2.4
2.5	0.99379	0.99396	0.99413	0.99430	0.99446	2.5
2.6	0.99534	0.99547	0.99560	0.99573	0.99585	2.6
2.7	0.99633	0.99644	0.99654	0.99663	0.99672	2.7
2.8	0.99744	0.99752	0.99760	0.99767	0.99774	2.8
2.9	0.99813	0.99819	0.99825	0.99831	0.99836	2.9
3.0	0.99865	0.99869	0.99874	0.99878	0.99882	3.0
3.1	0.99903	0.99906	0.99910	0.99913	0.99916	3.1
3.2	0.99931	0.99934	0.99936	0.99938	0.99940	3.2
3.3	0.99952	0.99953	0.99955	0.99957	0.99958	3.3
3.4	0.99966	0.99968	0.99969	0.99970	0.99971	3.4
3.5	0.99977	0.99978	0.99979	0.99979	0.99980	3.5
3.6	0.99984	0.99985	0.99985	0.99986	0.99986	3.6
3.7	0.99989	0.99990	0.99990	0.99990	0.99991	3.7
3.8	0.99993	0.99993	0.99993	0.99994	0.99994	3.8
3.9	0.99995	0.99995	0.99996	0.99996	0.99996	3.9

(continues)

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TABLE

Table II Cumulative Standard Normal Distribution (continued)

$$\Phi(z) = \int_{-\infty}^z \frac{1}{\sqrt{2\pi}} e^{-t^2/2} dt$$

z	0.05	0.06	0.07	0.08	0.09	z
0.0	0.51994	0.52392	0.52790	0.53188	0.53586	0.0
0.1	0.53962	0.54356	0.54749	0.55142	0.55534	0.1
0.2	0.59871	0.60257	0.60642	0.61026	0.61409	0.2
0.3	0.63683	0.64058	0.64431	0.64803	0.65173	0.3
0.4	0.67364	0.67724	0.68082	0.68438	0.68793	0.4
0.5	0.70884	0.71226	0.71566	0.71904	0.72240	0.5
0.6	0.74215	0.74537	0.74857	0.75175	0.75490	0.6
0.7	0.77337	0.77637	0.77935	0.78230	0.78523	0.7
0.8	0.80234	0.80510	0.80785	0.81057	0.81327	0.8
0.9	0.82894	0.83147	0.83397	0.83646	0.83891	0.9
1.0	0.85314	0.85543	0.85769	0.85993	0.86214	1.0
1.1	0.87493	0.87697	0.87900	0.88100	0.88297	1.1
1.2	0.89435	0.89616	0.89796	0.89973	0.90147	1.2
1.3	0.91149	0.91308	0.91465	0.91621	0.91773	1.3
1.4	0.92647	0.92785	0.92922	0.93056	0.93189	1.4
1.5	0.93943	0.94062	0.94179	0.94295	0.94408	1.5
1.6	0.95053	0.95154	0.95254	0.95352	0.95448	1.6
1.7	0.95994	0.96080	0.96164	0.96246	0.96327	1.7
1.8	0.96784	0.96856	0.96926	0.96995	0.97062	1.8
1.9	0.97441	0.97500	0.97558	0.97615	0.97670	1.9
2.0	0.97982	0.98030	0.98077	0.98124	0.98169	2.0
2.1	0.98422	0.98461	0.98500	0.98537	0.98574	2.1
2.2	0.98778	0.98809	0.98840	0.98870	0.98899	2.2
2.3	0.99061	0.99086	0.99111	0.99134	0.99158	2.3
2.4	0.99286	0.99305	0.99324	0.99343	0.99361	2.4
2.5	0.99461	0.99477	0.99492	0.99506	0.99520	2.5
2.6	0.99598	0.99609	0.99621	0.99632	0.99643	2.6
2.7	0.99702	0.99711	0.99720	0.99728	0.99736	2.7
2.8	0.99781	0.99788	0.99795	0.99801	0.99807	2.8
2.9	0.99841	0.99846	0.99851	0.99856	0.99861	2.9
3.0	0.99886	0.99889	0.99893	0.99897	0.99900	3.0
3.1	0.99918	0.99921	0.99924	0.99926	0.99929	3.1
3.2	0.99942	0.99944	0.99946	0.99948	0.99950	3.2
3.3	0.99960	0.99961	0.99962	0.99964	0.99965	3.3
3.4	0.99972	0.99973	0.99974	0.99975	0.99976	3.4
3.5	0.99981	0.99981	0.99982	0.99983	0.99983	3.5
3.6	0.99987	0.99987	0.99988	0.99988	0.99989	3.6
3.7	0.99991	0.99992	0.99992	0.99992	0.99992	3.7
3.8	0.99994	0.99994	0.99995	0.99995	0.99995	3.8
3.9	0.99996	0.99996	0.99996	0.99997	0.99997	3.9

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Table XIII Factors for Quality-Control Charts

n^a	\bar{X} Chart		R Chart		
	Factors for Control Limits		Factors for Central Line	Factors for Control Limits	
	A_1	A_2	d_2	D_3	D_4
2	3.760	1.880	1.128		
3	2.394	1.023	1.693	0	3.267
4	1.880	0.729	2.059	0	2.575
5	1.596	0.577	2.326	0	2.282
6	1.410	0.483	2.534	0	2.115
7	1.277	0.419	2.704	0	2.004
8	1.175	0.373	2.847	0.076	1.924
9	1.094	0.337	2.970	0.136	1.864
10	1.028	0.308	3.078	0.184	1.816
11	0.973	0.285	3.173	0.223	1.777
12	0.925	0.266	3.258	0.256	1.744
13	0.884	0.249	3.336	0.284	1.716
14	0.848	0.235	3.407	0.308	1.692
15	0.816	0.223	3.472	0.329	1.671
16	0.788	0.212	3.532	0.348	1.652
17	0.762	0.203	3.588	0.364	1.636
18	0.738	0.194	3.640	0.379	1.621
19	0.717	0.187	3.689	0.392	1.608
20	0.697	0.180	3.735	0.404	1.596
21	0.679	0.173	3.778	0.414	1.586
22	0.662	0.167	3.819	0.425	1.575
23	0.647	0.162	3.858	0.434	1.566
24	0.632	0.157	3.895	0.443	1.557
25	0.619	0.153	3.931	0.452	1.548
				0.459	1.541

^a $n > 25$; $A_1 = 3/\sqrt{n}$. n = number of observations in sample.

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Table III Percentage Points of the χ^2 Distribution*

ν	α	0.995	0.990	0.975	0.950	0.900	0.500	0.100	0.050	0.025	0.010	0.005
1		0.00+	0.00+	0.00+	0.00+	0.02	0.45	2.71	3.84	5.02	6.63	7.88
2		0.01	0.02	0.05	0.10	0.21	1.39	4.61	5.99	7.38	9.21	10.60
3		0.07	0.11	0.22	0.35	0.58	2.37	6.25	7.81	9.35	11.34	12.84
4		0.21	0.30	0.48	0.71	1.06	3.36	7.78	9.49	11.14	13.28	14.86
5		0.41	0.55	0.83	1.15	1.61	4.35	9.24	11.07	12.83	15.09	16.75
6		0.68	0.87	1.24	1.64	2.20	5.35	10.65	12.59	14.45	16.81	18.55
7		0.99	1.24	1.69	2.17	2.83	6.35	12.02	14.07	16.01	18.48	20.28
8		1.34	1.65	2.18	2.73	3.49	7.34	13.36	15.51	17.53	20.09	21.96
9		1.73	2.09	2.70	3.33	4.17	8.34	14.68	16.92	19.02	21.67	23.59
10		2.16	2.56	3.25	3.94	4.87	9.34	15.99	18.31	20.48	23.21	25.19
11		2.60	3.05	3.82	4.57	5.58	10.34	17.28	19.68	21.92	24.72	26.76
12		3.07	3.57	4.40	5.23	6.30	11.34	18.55	21.03	23.34	26.22	28.30
13		3.57	4.11	5.01	5.89	7.04	12.34	19.81	22.36	24.74	27.69	29.82
14		4.07	4.66	5.63	6.57	7.79	13.34	21.06	23.68	26.12	29.14	31.32
15		4.60	5.23	6.27	7.26	8.55	14.34	22.31	25.00	27.49	30.58	32.80
16		5.14	5.81	6.91	7.96	9.31	15.34	23.54	26.30	28.85	32.00	34.27
17		5.70	6.41	7.56	8.67	10.09	16.34	24.77	27.59	30.19	33.41	35.72
18		6.26	7.01	8.23	9.39	10.87	17.34	25.99	28.87	31.53	34.81	37.16
19		6.84	7.63	8.91	10.12	11.65	18.34	27.20	30.14	32.85	36.19	38.58
20		7.43	8.26	9.59	10.85	12.44	19.34	28.41	31.41	34.17	37.57	40.00
21		8.03	8.90	10.28	11.59	13.24	20.34	29.62	32.67	35.48	38.93	41.40
22		8.64	9.54	10.98	12.34	14.04	21.34	30.81	33.92	36.78	40.29	42.80
23		9.26	10.20	11.69	13.09	14.85	22.34	32.01	35.17	38.08	41.64	44.18
24		9.89	10.86	12.40	13.85	15.66	23.34	33.20	36.42	39.36	42.98	45.56
25		10.52	11.52	13.12	14.61	16.47	24.34	34.28	37.65	40.65	44.31	46.93
26		11.16	12.20	13.84	15.38	17.29	25.34	35.56	38.89	41.92	45.64	48.29
27		11.81	12.88	14.57	16.15	18.11	26.34	36.74	40.11	43.19	46.96	49.65
28		12.46	13.57	15.31	16.93	18.94	27.34	37.92	41.34	44.46	48.28	50.99
29		13.12	14.26	16.05	17.71	19.77	28.34	39.09	42.56	45.72	49.59	52.34
30		13.79	14.95	16.79	18.49	20.60	29.34	40.26	43.77	46.98	50.89	53.67
40		20.71	22.16	24.43	26.51	29.05	39.34	51.81	55.76	59.34	63.69	66.77
50		27.99	29.71	32.36	34.76	37.69	49.33	63.17	67.50	71.42	76.15	79.49
60		35.53	37.48	40.48	43.19	46.46	59.33	74.40	79.08	83.30	88.38	91.95
70		43.28	45.44	48.76	51.74	55.33	69.33	85.53	90.53	95.02	100.42	104.22
80		51.17	53.54	57.15	60.39	64.28	79.33	96.58	101.88	106.63	112.33	116.32
90		59.20	61.75	65.65	69.13	73.29	89.33	107.57	113.14	118.14	124.12	128.30
100		67.33	70.06	74.22	77.93	82.36	99.33	118.50	124.34	129.56	135.81	140.17

* ν = degrees of freedom.

Table IV Percentage Points of the t Distribution

ν	α	0.40	0.25	0.10	0.05	0.025	0.01	0.005	0.0025	0.001	0.0005
1		0.325	1.000	3.078	6.314	12.706	31.821	63.657	127.32	318.31	636.62
2		0.289	0.816	1.886	2.920	4.303	6.965	9.925	14.089	23.326	31.598
3		0.277	0.765	1.638	2.353	3.182	4.541	5.841	7.453	10.213	12.924
4		0.271	0.741	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5		0.267	0.727	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869
6		0.265	0.718	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7		0.263	0.711	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8		0.262	0.706	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9		0.261	0.703	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10		0.260	0.700	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11		0.260	0.697	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
12		0.259	0.695	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
13		0.259	0.694	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14		0.258	0.692	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140
15		0.258	0.691	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
16		0.258	0.690	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
17		0.257	0.689	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965
18		0.257	0.688	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
19		0.257	0.688	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
20		0.257	0.687	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
21		0.257	0.686	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
22		0.256	0.686	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792
23		0.256	0.685	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.767
24		0.256	0.685	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745
25		0.256	0.684	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725
26		0.256	0.684	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707
27		0.256	0.684	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.690
28		0.256	0.683	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674
29		0.256	0.683	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.659
30		0.256	0.683	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646
40		0.255	0.681	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551
60		0.254	0.679	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460
120		0.254	0.677	1.289	1.658	1.980	2.358	2.617	2.860	3.160	3.373
∞		0.253	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

Source: This table is adapted from *Biometrika Tables for Statisticians*, Vol. 1, 3rd edition, 1966, by permission of the Biometrika Trustees.

Table V Percentage Points of the F Distribution (continued)

v_1	$F_{0.05, v_1, v_2}$																		
	Degrees of freedom for the numerator (v_2)																		
v_2	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9	243.9	245.9	248.0	249.1	250.1	251.1	252.2	253.3	254.3
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.43	19.45	19.45	19.46	19.47	19.48	19.49	19.50
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.36
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.23	2.19	2.15	2.11	2.06	2.01
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.19	2.15	2.11	2.06	2.02	1.97
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.16	2.11	2.07	2.02	1.97	1.92
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.12	2.04	1.99	1.95	1.90	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.10	2.01	1.96	1.92	1.87	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.07	1.98	1.94	1.89	1.84	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	2.05	1.96	1.91	1.86	1.81	1.76
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	2.03	1.94	1.89	1.84	1.79	1.73
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	2.01	1.92	1.87	1.82	1.77	1.71
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.07	1.99	1.99	1.90	1.85	1.80	1.75	1.69
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13	2.06	1.97	1.97	1.88	1.84	1.79	1.73	1.67
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.04	1.96	1.96	1.87	1.82	1.77	1.71	1.65
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.10	2.03	1.94	1.94	1.85	1.81	1.75	1.70	1.64
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39
120	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.61	1.55	1.55	1.43	1.35	1.25
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00

(continued)