Marks scored:

KATHMANDU UNIVERSITY
End Semester Examination [C]
May/June, 2019

Level : B. E.
Year : II

Course : MATH 205
Semester : I

Exam Roll No. :

Time: 30 mins.

F. M. : 20

Registration No.:

Date 31 MAY 2019

SECTION "A"

[10 Q. × 1= 10 marks]

Fill in the blank space(s) with the most appropriate word(s) or symbol(s).

1. Shifting of origin without changing the direction of axes is called ....

2. Two diameters $y = mx + c_1$ and $y = m_1 x + c_2$ of a conic $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ are conjugate if ....

3. The condition for the line $\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$ to be perpendicular to the plane $ax + by + cz + d = 0$ is ....

4. The radius of the sphere $x^2 + y^2 + z^2 - 2x + 4y - 4z - 7 = 0$ is ....

5. Any point on the line $\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n} = r$ is ....

6. The direction cosines of a line equally inclined to the coordinate axes are................

7. The arc of a great circle drawn from a pole of a great circle to any point in its circumference is a ....

8. The equation of the tangent plane at (α, β, γ) of the sphere $x^2 + y^2 + z^2 = r^2$ is ....

9. The equation of the directrix of a conic $\frac{l}{r} = 1 + e\cos\theta$ is ....

10. The angle between the planes $x + 2y - 3z + 4 = 0$ and $2x + 5y + 4z + 1 = 0$ is................

SECTION "B"

[10 Q. × 1=10 marks]

Fill in the blank space(s), DO NOT TICK, by selecting the most appropriate answers from among the given ones.

11. The conic $\frac{l}{r} = 1 + \cos\theta$ represents a ....
[parabola; ellipse; hyperbola; circle]

12. If $S_1 = 0, S_2 = 0$ are two spheres, then $S_1 - S_2 = 0$ represents ....
[circle; sphere; plane; ellipse]

13. The straight line through (1,2,3) and parallel to the x-axis is................
[$y = 2, z = 3$; $x = 1, y = 2$; $x = 1, z = 3$; $x = 1, y = 2, z = 3$]

14. The two planes represented by $ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy = 0$ will be perpendicular to each other if ... ...
[$abc = 0$; $a + b = 0$; $a + b + c = 0$; $\frac{a}{b} = \frac{c}{f} = \frac{g}{h}$]

15. If $A_1B_1C_1$ is the polar triangle of a spherical triangle ABC, then b_1=
[π-B; B_1; π-B_1; π-b]

16. The line $\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$ lies in the plane $ax + by + cz + d = 0$ if $ax_1 + by_1 + cz_1 + d = 0$ and ....
[$al + bm + cn = 0$; $al + bm + cn > 0$; $al + bm + cn < 0$; $al + bm + cn \neq 0$]

17. The arc of the great circle intercepted between nearer pole and a point on the small circle is called ....
[spherical angle; spherical radius; pole; angular distance]

18. If the lines $x = -2y + 7, z = ky + 10$and $x = 5y - 1, z = 3y - 6$ are perpendicular to each other, then k =..............
[3; 1; 0; -2]

19. The locus of a point in three dimensional space which moves in such a way that its distance from a fixed point is always constant is called ....
[Cone; cylinder; plane; sphere]

20. The general equation of second degree $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents an ellipse if ... ... and $abc + 2fgh - af^2 - bg^2 - ch^2 \neq 0$.
[$h^2 - ab$=0; $h^2 + ab$=0; $h^2 - ab$ <0; $h^2 - ab$ >0]

KATHMANDU UNIVERSITY
End Semester Examination [C]
May/June, 2019

31 MAY 2019

Level : B. E.
Year : II
Time : 2 hrs. 30 mins.

Course : MATH 205
Semester : I
F. M. : 55

SECTION "C"
[4Q. × 7 = 28 marks]

1. Find the centre of the conic $3x^2 + 10xy + 3y^2 - 26x - 22y + 43 = 0$. Show that the conic is a hyperbola. Also, find the equations of the transverse and conjugate axes. [2+2+3]

OR

Find the condition that the straight line $lx + my + n = 0$ may touch the conic $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$. Hence find the equation of the tangent to the conic $x^2 + 4xy + 3y^2 + 5x - 6y + 3 = 0$ which are parallel to the line $x + 4y = 0$. [4+3]

2. Define skew lines and the line of shortest distance between them. Find the shortest distance between the lines $\frac{x-3}{2} = \frac{y-4}{3} = \frac{z-5}{4}$ and $\frac{x-4}{3} = \frac{y-5}{4} = \frac{z-7}{5}$. Also, find the equation of the shortest distance. [1+4+2]

3. Define a great and a small circle of a sphere. Prove that the equation of the plane containing the line $\frac{y}{b} + \frac{z}{c} = 1, x = 0$ and parallel to the line $\frac{x}{a} - \frac{z}{c} = 1, y = 0$ is $\frac{x}{a} - \frac{y}{b} - \frac{z}{c} + 1 = 0$. If 2d is the shortest distance between the given lines, prove that $\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{d^2}$. [1+3+3]

4. For any spherical triangle ABC, prove that $cosA = \frac{cosa - cosb cosc}{sinb sinc}$ where the symbols have their usual meanings. Also, show that the sides of a polar triangle are respectively supplements of the angles and sides of the primitive triangle. [4+3]

SECTION "D"
[9Q. × 3 = 27 marks]

5. Find the angle through which the axes may be turned so that equation $Ax + By + C = 0$ may be reduced to the form X = constant. Also, determine the value of this constant.

6. Prove that the locus of the pole of a chord which subtends a constant angle 2β at the focus of the conic $\frac{l}{r} = 1 + e cos\theta$ is $\frac{l sec\beta}{r} = 1 + e sec\beta cos\theta$.

OR

If PSP_1 and QSQ_1 are two perpendicular focal chords passing through the focus S of a conic $\frac{l}{r} = 1 + e cos\theta$, prove that $\frac{1}{PP_1} + \frac{1}{QQ_1}$ is constant.

7. Find the equation of the chord joining the two points (r_1, θ_1) and (r_2, θ_2) on the conic $\frac{l}{r} = 1 + e cos\theta$.

8. Find the equations of the line passing through a point (α, β, γ) and at right angles to the lines $\frac{x}{l_1} = \frac{y}{m_1} = \frac{z}{n_1}$, and $\frac{x}{l_2} = \frac{y}{m_2} = \frac{z}{n_2}$.

9. Find the length of an arc **ab** of a small circle, if an arc **AB** of a corresponding great circle is given.

OR

For the spherical triangle ABC, prove that $sin\frac{A}{2} = \sqrt{\frac{sin(s-b)sin(s-c)}{sinbsinc}}$.

10. Find the equation of the chord of the conic $x^2 + 4xy + y^2 - 2x + 2y - 6 = 0$ whose middle point is $(-1,2)$.

11. Show that the lines $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z+2}{1}$ and $x = \frac{y-7}{-3} = \frac{z+7}{2}$ are coplanar. Also, find the equation of the plane containing them.

12. Obtain the equation of the sphere which passes through the three points (1,0,0), (0,1,0), (0,0,1) and has its radius as small as possible.

13. Obtain the equation of the plane through the intersection of the plane $x + 2y + 3z + 4 = 0$ and $4x + 3y + 2z + 1 = 0$ and passes through the origin.