KATHMANDU UNIVERSITY
End Semester Examination
March, 2025

Marks Scored:

Level : B.E.		Course : MATH 205
Year : II		Semester : I
Exam Roll No. :	Time: 30 mins.	F. M. : 20
Registration No.:		Date : 21 MAR 2025

SECTION "A"
[10 Q. × 1 = 10 marks]

Fill in the blank space(s) by writing the most appropriate word(s) or symbol(s).

1. If the origin is transferred to $(-1,\ 2)$ axes are parallelly shifted, then the transformed equation of $x^2 - y^2 + 2x + 4y = 0$ is ____________________.

2. The equation of directrix of the conic $\frac{\ell}{r} = 1 + e\cos\theta$ is ____________________.

3. An equation of conic section will be a parabola if it has two ______________ asymptotes.

4. The points $(1, -1,\ 2)$ and $(3, 2, -1)$ lie on the ______________ side(s) of the plane $x + 2y - 3z - 4 = 0$.

5. The equation of a line through $(-1, 3, 2)$ and perpendicular to the plane $x + 2y + 2z = 3$ is ____________________.

6. Lines which are not parallel and which do not intersect at a point are called ____________.

7. The plane section of the sphere $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$ represents a great circle if radius of the circle is ______________ radius of the sphere.

8. Equation of tangent plane at (α, β, γ) of a sphere $x^2 + y^2 + z^2 = r^2$ is ____________________.

9. Let ABC be a spherical triangle and $A'B'C'$ be its polar triangle, then $A' =$ __________, where symbols have usual meanings.

10. The number of great circle through the given two points of the sphere is ____________ where the centre of the sphere and given points are not colinear.

SECTION "B"

[10 Q.× 1 = 10 marks]

Fill in the blank space(s), **DO NOT TICK**, by selecting the most appropriate answers from among the given ones.

11. The angle through which the axes must be rotated to remove the term containing xy in $3x^2 + 2xy + 3y^2 - \sqrt{2}x = 0$ is ____________.
[π, $\frac{\pi}{2}$, $\frac{\pi}{4}$, 0

12. The cord of contact of point (r', α) with respect to the conic $\frac{\ell}{r} = 1 + e\cos\theta$ is ____________________.
[$\frac{\ell}{r} = \cos(\theta - \alpha) + e\cos\theta$; $\left(\frac{\ell}{r'} - \cos(\theta - \alpha)\right)\left(\frac{\ell}{r} - \cos\theta\right) = e\cos\theta$;
$\left(\frac{\ell}{r'} - \cos\alpha\right)\left(\frac{\ell}{r} - \cos\theta\right) = e\cos\theta$; $\left(\frac{\ell}{r'} - e\cos\alpha\right)\left(\frac{\ell}{r} - e\cos\theta\right) = \cos(\theta - \alpha)$]

13. What curve does the equation $2x^2 + 5xy + 3y^2 + 5x - 7y + 2 = 0$ represent? ____________.
[Ellipse, Hyperbola, Parabola, Pair of straight lines]

14. The locus of the middle points of a system of parallel chords of a conic is called ____________.
[Conjugate axis, Directrix, Diameter, Polar line]

15. Direction cosines of a line equally inclined to the co-ordinate axes are ____________.
[1, 1, 1; $\frac{1}{3}, \frac{1}{3}, \frac{1}{3}$; $-\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}$; $\pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}$]

16. The line $\frac{x-x_1}{\ell} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$ may perpendicular to the plane $ax + by + cz + d = 0$, if____________________.
[$\frac{a}{\ell} = \frac{b}{m} = \frac{c}{n}$, $\frac{a}{x_1} = \frac{b}{y_1} = \frac{c}{z_1}$, $a\ell + bm + cn = 0$, $ax_1 + by_1 + cz_1 = 0$]

17. The number of arbitrary constants in the equation of a line $\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$ are____________.
[(6, 5, 4, 3]

18. Let $S_1: x^2 + y^2 + z^2 + 2u_1x + 2v_1y + 2w_1z + d_1 = 0$ and $S_2: x^2 + y^2 + z^2 + 2u_2x + 2v_2y + 2w_2z + d = 0$ be two spheres then $S_1 - S_2 = 0$ is a____________.
[Sphere, Plane , Circle, Ellipse]

19. The inclination of two arcs of great circles at their points of intersection on the surface of the sphere is called ____________________.
[Angular distance, Spherical radius, Spherical angle, Shortest arc]

20. The sum of three sides of a spherical triangle is ____________.
[Equal to 2π, Less or equal to 2π, Less than 2π, Greater than 2π]

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F. M. : 55

SECTION "C"
[4 Q. × 7 = 28 marks]

1. Define pole and polar of conic section. Find the equation of the polar of (r', α) with respect to the conic $\frac{\ell}{r} = 1 + ecos\theta$. If S be the focus and P and Q be two points on a conic such that the angle PSQ is constant, prove that the locus of the point of intersection of the tangents at P andf Q is a conic section whose focus is S. [1+3+3]

OR

Find the equation of asymptotes of the conic $\frac{\ell}{r} = 1 + ecos\theta, e > 1$. Let PSP' and QSQ' be two perpendicular focal chords of a conic $\frac{\ell}{r} = 1 + ecos\theta$; prove that $\frac{1}{PP'} + \frac{1}{QQ'}$ is constant, S being the focus of the conic. [4+3]

2. Transform the following straight line into symmetrical form: $a_1x + b_1y + c_1z + d_1 = 0 = a_2x + b_2y + c_2z + d_2$. What is the line of the shortest distance? Find the length of the line of the shortest distance between the lines $\frac{x-3}{-1} = \frac{y-4}{2} = \frac{z+2}{1}$ and $\frac{x-1}{1} = \frac{y+7}{3} = \frac{z+2}{2}$. [3+1+3]

3. Derive the equation of the sphere when the ends of its diameter are given. A sphere of radius k passes through origin and meets the axes in A, B, C. Prove that the foot of the perpendicular from origin to the plane ABC lies on the surface

$$(x^2 + y^2 + z^2)^2(x^{-2} + y^{-2} + z^{-2}) = 4k^2.$$ [3+4]

4. Prove that the sines of the angles of a spherical triangle are proportional to the sines of the opposite sides. Show that: $\cos\frac{1}{2}A = \sqrt{\frac{\sin s \sin(s-a)}{\sin b \sin c}}$. [4+3]

P.T.O.

SECTION "D"

[9Q. × 3 = 27 marks]

5. Show that the transformation which converts $\frac{X^2}{p} + \frac{Y^2}{q}$ into $ax^2 + 2hxy + by^2$ will convert $\frac{X^2}{p-\lambda} + \frac{Y^2}{q-\lambda}$ into $\frac{ax^2+2hxy+by^2-\lambda(ab-h^2)(x^2+y^2)}{1-\lambda(a+b)+\lambda^2(ab-h^2)}$.

OR

What does the equation $(a-b)(x^2+y^2) - 2abx = 0$ becomes if the origin be moved to the point $(\frac{ab}{a-b},\ 0)$?

6. If possible, find the centre of $13x^2 - 18xy + 37y^2 + 2x + 14y - 2 = 0$ and its equation referred to the centre.

7. Find the equation of the chord of the conic $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$, whose middle point is $P(x_1, y_1)$.

8. Show that the lines $x = \frac{y-2}{2} = \frac{z+3}{3}$ and $\frac{x-2}{2} = \frac{y-6}{3} = \frac{z-3}{4}$ are coplanar and find their plane and point of intersection.

9. Find the distance of the point $(-2,1,5)$ from the line through $(2,3,5)$ whose direction cosines are proportional to $2,-3,\ 6$.

10. Determine the centre and radius of the sphere having the circle $x^2 + y^2 + z^2 = 9,\quad x - 2y + 2z = 5$ as a great circle.

11. Prove that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ are coplanar. Find the equation of the plane in which they lie.

12. Show that the equation $6x^2 + 4y^2 - 10z^2 + 3yz + 4zx - 11xy = 0$ represents a pair of perpendicular planes, find their equations.

13. Find the length of arc ab of small circle if the arc AB of the corresponding great circle is given.

OR

Prove that the sides of the angles of a polar triangle are respectively supplements of the sides and angles of the primitive triangle.