

KATHMANDU UNIVERSITY
End Semester Examination
March/April, 2017

Marks Scored:

Level : B. E.

Year : II

Exam Roll No. :

Time : 30 mins.

Course : MATH 205

Semester : I

F. M. : 20

Registration No. :

Date : MAR 27 2017

SECTION "A"

[10 Q. \times 1 = 10 marks]

Fill in the blank space (s) by most appropriate word (s) or symbol (s).

1. If the origin is transferred to (α, β) and the axes are turned through an angle θ , then the transformation will be given by the relations $x =$ _____,
 $y =$ _____.
2. When the axis of the conic $\frac{\rho}{r} = 1 + e \cos \theta$ makes an angle α with an initial line, the equation of the conic becomes _____.
3. Two points such that the polar of each with respect to a conic passes through the other are called _____ points.
4. The points $(1, -1, 2)$ and $(3, 2, -1)$ lie on the _____ side(s) of the plane $x + 2y - 3z - 4 = 0$.
5. If the line $\frac{x-2}{2} = \frac{y+3}{5} = \frac{z-5}{k}$ is parallel to the plane $2x - 3y + z = 3$, then $k =$ _____.
6. Lines which are not parallel and which do not intersect at a point are called _____.
7. The equation $ax^2 + ay^2 + az^2 + 2ux + 2vy + 2wz + d = 0$, $a \neq 0$ represents a sphere with radius (r) being _____.
8. The equation of tangent plane to a sphere $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz = 0$ at $(0, 0, 0)$ is _____.
9. For a spherical triangle ABC and its polar triangle $A'B'C'$, A' is _____.
10. All those great circles which pass through the poles of a given circle are called _____ to the given circle.

SECTION "B"

[10 Q. \times 1 = 10 marks]

Fill in the blank space (s) by choosing the most appropriate answer from among the given ones.
Do not tick the answers.

11. The equation of the curve $x + y + 3 = 0$ referred to new parallel axes through $(-1, -2)$ becomes _____.
[$x + y = 0$, $x + y - 3 = 0$, $x + y + 3 = 0$, $y - x + 3 = 0$]

12. The polar of the point (r', α) with respect to the conic $\frac{\ell}{r} = 1 + e \cos \theta$ is

$$\frac{\ell}{r} = \cos(\theta - \alpha) + e \cos \theta; \quad \left(\frac{\ell}{r'} - \cos(\theta - \alpha)\right) \left(\frac{\ell}{r} - \cos \theta\right) = e \cos \theta;$$

$$\left(\frac{\ell}{r'} - \cos \alpha\right) \left(\frac{\ell}{r} - \cos \theta\right) = e \cos \theta; \quad \left(\frac{\ell}{r'} - e \cos \alpha\right) \left(\frac{\ell}{r} - e \cos \theta\right) = \cos(\theta - \alpha)$$

13. The equation $x^2 + y^2 - 2xy - 2x - 1 = 0$ represents _____.
 [Ellipse, Hyperbola, Parabola, Circle]

14. The locus of middle points of a system of parallel chords of a conic is called _____.
 [Conjugate axis, Directrix, Diameter, Polar line]

15. Direction cosines of a line having the projections 6, 2, 3 on the co-ordinate axes are _____.
 [6, 2, 3; 12, 4, 6; $\frac{6}{7}, \frac{2}{7}, \frac{3}{7}$; $\frac{6}{49}, \frac{2}{49}, \frac{3}{49}$]

16. If the line $\frac{x-x_1}{\ell} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$ is normal to the plane $ax + by + cz + d = 0$, then _____.
 [$\frac{a}{\ell} = \frac{b}{m} = \frac{c}{n}$, $\frac{a}{x_1} = \frac{b}{y_1} = \frac{c}{z_1}$, $a\ell + bm + cn = 0$, $ax_1 + by_1 + cz_1 = 0$]

17. The line $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12}$ intersects the plane $x - y + z = 5$ at _____.
 [(2, -1, 2), (4, 0, -1), (0, -2, -3), (1, -1, -1)]

18. The plane $\ell x + my + nz = p$ touches the sphere $x^2 + y^2 + z^2 = a^2$ if $p =$ _____.
 [a , $\sqrt{\ell^2 + m^2 + n^2}$, $a\sqrt{\ell^2 + m^2 + n^2}$, $a^2\sqrt{\ell^2 + m^2 + n^2}$]

19. The inclination of two arcs of great circles at their points of intersection on the surface of the sphere is called _____.
 [Angular distance, Spherical radius, Spherical angle, Shortest arc]

20. The arc of a great circle drawn from a pole of a great circle to any point in its circumference is a _____.
 [Circle, Semi-circle, Quadrant, Secondary]

Level : B. E.
Year : II
Time : 2 hrs. 30 mins.

Course : MATH 205
Semester : I
F. M. : 55

SECTION "C"

[4 Q. × 7 = 28 marks]

1. Define the chord of contact to a curve. Find the equation of the chord of contact of (r', α) with respect to the conic $\frac{\rho}{r} = 1 + e \cos \theta$. Also, if PSP' and QSQ' are two perpendicular focal chords of a conic $\frac{\rho}{r} = 1 + e \cos \theta$, prove that $\frac{1}{PP'} + \frac{1}{QQ'}$ is constant, S being the focus of the conic. [1+3+3]

OR

Find the axis and latus rectum of the parabola given by $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$. Also, prove that the lengths of semi-axes of the central conic $ax^2 + 2hxy + ay^2 = d$ are $\sqrt{\frac{d}{a+d}}$ and $\sqrt{\frac{d}{a-d}}$ respectively and their combined equation is $x^2 - y^2 = 0$.

[4+3]

2. Define the line of the shortest distance? Find the length of the line of the shortest distance between the lines $\frac{x-3}{-1} = \frac{y-4}{2} = \frac{z+2}{1}$ and $\frac{x-1}{1} = \frac{y+7}{3} = \frac{z+2}{2}$. Also, find the equation of the line of the shortest distance. [1+4+2]
3. Derive the equation of the sphere for the given ends of its diameter. If a sphere of radius k passes through origin and meets the axes in A, B, C , then prove that the foot of the perpendicular from origin to the plane ABC lies on the surface $(x^2 + y^2 + z^2)^2(x^{-2} + y^{-2} + z^{-2}) = 4k^2$. [3+4]
4. Prove that the sines of the angles of a spherical triangle are proportional to the sines of the opposite sides. Also, show that: $\sin \frac{1}{2}A = \sqrt{\frac{\sin(s-b) \sin(s-c)}{\sin b \sin c}}$. [4+3]

SECTION "D"

[9 Q. × 3 = 27 marks]

5. Translate the axes so as to change the equation $3x^2 - 2xy + 4y^2 + 8x - 10y + 8 = 0$ into an equation with linear terms missing.

OR

Find the angle through which the axes must be rotated to remove the term containing xy in $3x^2 + 2xy + 3y^2 - \sqrt{2}x = 0$. Also, find the transformed equation.

6. Derive the condition that the line $\frac{\rho}{r} = \alpha \cos \theta + \beta \sin \theta$ may touch the conic $\frac{\rho}{r} = 1 + e \cos \theta$.
7. Find the middle point of chord $9x - 4y = 14$ relative to the conic $2x^2 + xy - 3y^2 = 1$.

8. Find the equation of the plane which contains the line $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z-3}{4}$ and is perpendicular to the plane $x + 2y + z = 12$.
9. Prove that the lines $x = ay + b, z = cy + d$ and $x = a'y + b', z = c'y + d'$ are perpendicular if $aa' + cc' + 1 = 0$.
10. Determine the centre and radius of the sphere having the circle $x^2 + y^2 + z^2 = 9$, $x - 2y + 2z = 5$ as a great circle.
11. Prove that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ are coplanar. Also, find the equation of the plane in which they lie.
12. Show that the equation $6x^2 + 4y^2 - 10z^2 + 3yz + 4zx - 11xy = 0$ represents a pair of perpendicular planes and also find their equations.
13. Find the length of arc ab of small circle if the arc AB of the corresponding great circle is given.

OR

Prove that the sides of the angles of a polar triangle are respectively supplements of the sides and angles of the primitive triangle.