KATHMANDU UNIVERSITY
End-Semester Examination [C]
June 2018

Marks Scored:

Level : B.E.
Year : II

Course : MATH 205
Semester: I

Exam Roll No.:

Time: 30 mins

F.M. : 20

Registration No:

Date JUN 11 2018

SECTION "A"
[10 Q.×1=10 marks]

Fill in the blank space (s) by writing the most appropriate word(s) or symbol(s).

1. If any change of axes, without change of origin $ax^2 + 2hxy + by^2$ become $AX^2 + 2HXY + BY^2$, the axes in each case being rectangular, then $a + b$ =..... and $ab - h^2$ =..... .

2. The locus of the middle points of a system of parallel chords of conic is called.....

3. If the line $\frac{x-2}{2} = \frac{y+3}{5} = \frac{z-5}{k}$ is parallel to the plane $2x - 3y + z = 3$, then the value of k is .....

4. The equation of the sphere passing through the origin and the points $(a, 0,0)$, $(0, b, 0)$ and $(0,0, c)$ is .....

5. The equation of the x-axis is .....

6. Length of projection of the line segment joining (x_1, y_1, z_1) and (x_2, y_2, z_2) on a line with directions cosines is l, m, n is .....

7. The sum of three sides of any spherical triangle is less than .....

8. The spheres $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$ and $x^2 + y^2 + z^2 + 2u_1x + 2v_1y + 2w_1z + d_1 = 0$ intersect orthogonally if $2uu_1 + 2vv_1 + 2ww_1$ =.....

9. The equation of the tangent to the conic $\frac{l}{r} = 1 + ecos\theta$ at the point α on it is .....

10. The equation of the plane through $(1, 2, 3)$ and parallel to $3x + 4y - 5z = 0$ is.....

SECTION "B"

[10 Q.×1=10 marks]

Fill in the blank space(s), DO NOT TICK, by selecting the most appropriate answers from among the given ones.

11. The conic $\frac{l}{r} = 1 + e\cos\theta$ represents a parabola if .....
[e = 0; e = 1; e < 1; e >1]

12. If $S_1 = 0$, $S_2 = 0$ are two spheres then $S_1 + \lambda S_2 = 0$ represents ... ...
[circle, sphere, line, ellipse]

13. If $A_1B_1C_1$ is the polar triangle of a spherical triangle ABC, then a_1=... ... ...
[π-A; A_1; π-A_1; π-a]

14. If the line makes angles α, β, and γ with the axes, then $\cos 2\alpha + \cos 2\beta + \cos 2\gamma =$ .....
[-2; -1; 1; 2]

15. The general equation of second degree represents hyperbola if ... ...
[$h^2 - ab$=0; $h^2 + ab$=0; $h^2 - ab$ <0; $h^2 - ab$ >0]

16. The line $\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$ lies in the plane $Ax + By + Cz + D = 0$ if $aA + bB + cC + D = 0$ and .....
[$Ax_1 + By_1 + Cz_1 + D = 0$; $Ax_1 + By_1 + Cz_1 + D > 0$;
$Ax_1 + By_1 + Cz_1 + D < 0$; $Ax_1 + By_1 + Cz_1 + D \neq 0$]

17. The inclination of two arcs of great circles at their points of intersection on the surface of the sphere is called : .....
[Spherical angle; Spherical radius; Pole; Angular distance]

18. If the lines $x = ky + 7$, $z = 3y + 10$ and the line $x = 5y - 1, z = 3y - 6$ are perpendicular, then the value of k =.....
[2; 1; 0; -2]

19. The equations to the straight line through (a, b, c) parallel to the x-axis are.....
[$y = b, z = c$; $x = a, y = b$; $x = a, z = c$; $x = a, y = b, z = c$]

20. The smallest radius of the sphere passing through (1,0,0), (0,1,0) and (0,0,1) is .....
[$\sqrt{\frac{3}{5}}$; $\sqrt{\frac{3}{8}}$; $\sqrt{\frac{2}{3}}$; $\sqrt{\frac{5}{12}}$]

KATHMANDU UNIVERSITY
End Semester Examination [C]
June 2018

JUN 11 2018

Level : B.E.
Year : II
Time : 2 hrs. 30 mins.

Course : MATH 205
Semester: II
F.M. : 55

SECTION "C"
[4 Q. × 7 = 28 marks]

1. What conic does the equation $13x^2 - 18xy + 37y^2 + 2x + 14y - 2 = 0$ represent? If possible, find its centre and its equation referred to the centre. [3+4]

OR

Define pole and polar with respect to the conic. Determine the equation of polar with respect to the conic represented by $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$. [2+5]

2. When will two given lines be coplanar? Write the condition for two straight lines in symmetrical form to be coplanar. Prove that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $4x - 3y + 1 = 0 = 5x - 3z + 2$ are coplanar. Also, find the point of intersection. [1+1+3+2]

3. When will a plane be tangent to a sphere? What will happen if a plane passes through the centre of the circle? Find the equation of the sphere through the circle $x^2 + y^2 + z^2 = 1$, $2x + 4y + 5z = 6$ and touches the plane $z = 0$. [1+1+5]

4. For any spherical triangle ABC, prove that (i) $a + b + c < 2\pi$ and (ii) $\cos A = \frac{\cos a - \cos b \cos c}{\sin b \sin c}$ where the symbols have their usual meanings. [3+4]

SECTION "D"
[9 Q. × 3 = 27 marks]

5. If the axes be turned through an angle $\tan^{-1}2$, what does the equation $4xy - 3x^2 = a^2$ become?

6. Prove that the locus of the middle point of focal chords of conic section is a conic section.

OR

If a chord PQ of a conic whose eccentricity is e and semi latus rectum is l subtends a right angle at the focus S, prove that $\left(\frac{1}{SP} - \frac{1}{l}\right)^2 + \left(\frac{1}{SQ} - \frac{1}{l}\right)^2 = \frac{e^2}{l^2}$.

7. Find the condition that the line $\frac{l}{r} = A\cos\theta + B\sin\theta$ may touch the conic $\frac{l}{r} = 1 + e\cos\theta$.

8. Find the point where the line joining $(2,1,3), (4,-2,5)$ cuts the plane $2x + y - z - 3 = 0$.

9. Find the length of an arc **ab** of a small circle, if an arc **AB** of a corresponding great circle is given.

OR

For the spherical triangle, find $sin\frac{A}{2} = \sqrt{\frac{sin(s-b)sin(s-c)}{sinb\ sinc}}$.

10. Find the condition that the pair of lines, whose equation is $Ax^2 + 2Hxy + By^2 = 0$ may be conjugate diameters of the conic $ax^2 + 2hxy + by^2 = 1$.

11. Find the shortest distance between the lines $\frac{x}{2} = \frac{y}{-3} = \frac{z}{1}$ and $\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z+2}{2}$.

12. If any tangent plane to the sphere $x^2 + y^2 + z^2 = r^2$ makes intercepts a, b, c on the coordinate axes, prove that $a^{-2} + b^{-2} + c^{-2} = r^{-2}$.

13. The plane $lx + my = 0$ is rotated about its line of intersection with the plane $z = 0$ through an angle α. Prove that the equation of the plane in its new position is $lx + my \pm z \tan\alpha \sqrt{l^2 + m^2} = 0$.