

KATHMANDU UNIVERSITY
End Semester Examination
July/August, 2024

Level : B.E.
Year : II
Time : 2 hrs. 30mins.

02 AUG 2024

Course : MATH 205
Semester : I
F. M. : 55

SECTION "C"

[4 Q. × 7 = 28 marks]

1. What is the chord of contact? Find the equation of the chord of contact of (r_1, θ_1) with respect to the conic $\frac{l}{r} = 1 + e \cos \theta$. Show that in any conic, the semi-latus rectum is a harmonic mean between the segments of any focal chord. [1+3+3]

OR

Find the axis and latus rectum of the parabola given by $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$. Find the centre of the conic section $2x^2 + 5xy - 3y^2 - x - 4y + 6 = 0$ and its equation when transformed to the centre. [4+3]

2. Define the shortest distance. Find the magnitude of the line of the shortest distance between the lines $\frac{x}{4} = \frac{y+1}{3} = \frac{z-2}{2}$ and $5x - 2y - 3z + 6 = 0 = x - 3y + 2z - 3$. Also, find the equation of the line of the shortest distance. [1+4+2]
3. Derive the equation of the sphere when the ends of its diameter are given. Find the equations of the sphere through the circle $x^2 + y^2 + z^2 = 5$, $x + 2y + 3z = 3$ and touching the plane $4x + 3y = 15$. [3+4]
4. Define spherical angle. Prove that the sides of the angles of a polar triangle are respectively supplements of the angles and sides of the primitive triangle. Show that: $\sin \frac{1}{2}A = \sqrt{\frac{\sin(s-b) \sin(s-c)}{\sin b \sin c}}$. [1+3+3]

SECTION "D"

[9 Q. × 3 = 27 marks]

5. Find the equation of the tangent at the point, whose vectorial angle is θ_1 of the conic $\frac{l}{r} = 1 + e \cos \theta$.
6. Define conjugate lines and then show that the polar of $(1, 2)$ and $(-2, 3)$ are conjugate with respect to the conic $2x^2 + 6xy + y^2 + 4x - 2y + 8 = 0$.

7. Find the angle through which the axes must be rotated to remove the term containing xy in $3x^2 + 2xy + 3y^2 - \sqrt{2}x = 0$. Also, find the transformed equation.

OR

By a suitable rotation, if $ux + vy$ becomes $VX + UY$, where u and v are independent of x and y ; prove that $u^2 + v^2 = U^2 + V^2$.

8. Show that the equation $2x^2 - 6y^2 - 12z^2 + 18yz + 2zx + xy = 0$ represents a pair of planes, find the angle between them.
9. Prove that the lines $x = ay + b$, $z = cy + d$ and $x = a'y + b'$, $z = c'y + d'$ are perpendicular if $aa' + cc' + 1 = 0$.
10. Derive the condition for lines $\frac{x-x_1}{l_1} = \frac{y-y_1}{m_1} = \frac{z-z_1}{n_1}$ and $\frac{x-x_2}{l_2} = \frac{y-y_2}{m_2} = \frac{z-z_2}{n_2}$ to be coplanar.
11. Find the equation of the sphere which passes through the circle $x^2 + y^2 = 4, z = 0$ and is cut by the plane $x + 2y + 2z = 0$ is a circle of radius 3.
12. Prove that the tangent planes to the spheres $x^2 + y^2 + z^2 + 2u + 2vy + 2wz + d = 0$ and $x^2 + y^2 + z^2 + 2u_1x + 2v_1y + 2w_1z + d_1 = 0$ at any common point are at right angles if $2uu_1 + 2vv_1 + 2ww_1 = d + d_1$.
13. Find the length of arc ab of small circle if the arc AB of the corresponding great circle is given.

OR

Prove that the sum of three sides of a spherical triangle is less than the circumference of a great circle.

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SECTION "A"
[10 Q. × 1 = 10 marks]

Fill in the blank space(s) by most appropriate words or symbol(s):

1. If the origin is transferred to $(-h, -k)$ and the axes are turned through an angle θ , then the transformation will be given by the relations $x = \underline{\hspace{2cm}}$,
 $y = \underline{\hspace{2cm}}$.
2. The polar of (r', α) with respect to the conic $\frac{l}{r} = 1 + e \cos \theta$ is given by
 $\underline{\hspace{2cm}}$.
3. When the axis of the conic $\frac{\rho}{r} = 1 + e \cos \theta$ makes an angle π with an initial line, the equation of the conic becomes $\underline{\hspace{2cm}}$.
4. Two points such that the polar of each with respect to a conic passes through the other are called $\underline{\hspace{2cm}}$ points.
5. If the line $\frac{x-3}{2} = \frac{y+2}{5} = \frac{z-7}{k}$ is parallel to the plane $2x - 4y + z = 3$, then $k = \underline{\hspace{2cm}}$.
6. Line $\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$ and plane $ax + by + cz + d_2 = 0$ are parallel if
 $\underline{\hspace{2cm}}$.
7. The radius of the sphere $kx^2 + ky^2 + kz^2 + 2ux + 2vy + 2wz + d = 0$, $k \neq 0$, is $r = \underline{\hspace{2cm}}$.
8. If the radius of the circle is less than the radius of the sphere, the circle is called
 $\underline{\hspace{2cm}}$.
9. The extremities of the axis of any circle of a sphere are called $\underline{\hspace{2cm}}$ of the circle.
10. All those great circles which pass through the poles of a given circle are called
 $\underline{\hspace{2cm}}$ to the given circle.

SECTION "B"
[10 Q. × 1 = 10 marks]

Fill in the blank space(s), **DO NOT TICK**, by selecting the most appropriate answers from among the given ones.

11. For the removal of the xy term from $ax^2 + 2hxy + by^2$, the rectangular axes must be turned through the angle θ satisfying _____.
[$\tan 2\theta = \frac{h}{a-b}$, $\tan \theta = \frac{h}{2(a-b)}$, $\tan 2\theta = \frac{2h}{a-b}$, $\tan \theta = \frac{2h}{a+b}$]
12. The conic $\frac{\rho}{r} = e \cos \theta$ represents a hyperbola if _____.
[$e = 0$, $e < 1$, $e = 1$, $e > 1$]
13. If $\alpha_1, \alpha_2, \alpha_3$ be the angles that a line makes with the co-ordinate axes, then the value of $\cos^2 \alpha_1 + \cos^2 \alpha_2 + \cos^2 \alpha_3 - 1$ will be _____.
[-1, 0, 1, 2]
14. The locus of the middle points of a system of parallel chords of a conic is called _____.
[conjugate axis, directrix, diameter, polar line]
15. Two planes represented by $ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy = 0$ will be perpendicular to each other if
[$a = b + c$, $a + b + c = 0$, $a = bc$, $abc = 1$]
16. Direction cosines of a line normal to the plane $x - y + z = 3$ are proportional to _____.
[1, -1, 1, 1, -1, 3, $\frac{1}{3}, -\frac{1}{3}, \frac{1}{3}$, $\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$]
17. The equations to the straight lines through (a, b, c) parallel to the x-axis are _____.
[$x = a, y = b$, $x = b, y = c$, $y = b, z = c$, $z = a, x = b$]
18. If $S_1 = 0$ and $S_2 = 0$ are two spheres then $S_1 - S_2 = 0$ represents _____.
[Circle, Cylinder, Sphere, Straight Line]
19. The sum of three sides of a spherical triangle is _____.
[π , less than 2π , 2π , greater than 2π]
20. The inclination of two arcs of great circles at their points of intersection on the surface of a sphere is called _____.
[circle, semi-circle, quadrant, secondary]