KATHMANDU UNIVERSITY
End Semester Examination
June/July, 2023

Marks Scored:

Level : B.E.
Year : II

Course : MATH 205
Semester : I

Exam Roll No. :

Time: 30 mins.

F. M. : 20

Registration No.:

Date 30 JUN 2023

SECTION "A"
[10 Q. × 1 = 10 marks]

Fill in the blank space (s) by most appropriate word (s) or symbol (s).

1. The equation of the curve $y = x^2 + 5$ referred to new parallel axes through (2, 5) becomes _____.

2. The tangent at the point whose vectorial angle is α of the conic $\frac{\ell}{r} = 1 + e\cos\theta$ is _____.

3. The locus of the middle points of a system of parallel chords of a conic is called _____.

4. If α, β, γ be the angles which a line makes with the co-ordinate axes, then $\sin^2\alpha + \sin^2\beta + \sin^2\gamma =$ _____.

5. Angle between the pair of planes $2x - 3y - 3z = 0$ and $3x - 2y + 4z = 0$ is _____.

6. If the line $\frac{x-2}{1} = \frac{y+3}{3} = \frac{z-5}{k}$ is parallel to the plane $2x - 3y + z = 3$, then $k =$ _____.

7. Lines which are not parallel and which do not intersect at a point are called _____.

8. Any equation of the type $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$ always represents a sphere having length of radius is _____.

9. The equation of the sphere described on the joint of (a, b, c) and $(0, 0, 0)$ as diameter is _____.

10. The arc of a great circle drawn from a pole of great circle to any point in its circumference is a _____.

SECTION "B"

[10 Q.× 1 = 10 marks]

Fill in the blank space(s) by selecting the most appropriate answer from among the given ones. (*Do not tick the answer*).

11. If the axes be rotated through an angle θ so that the expression $x^2 + xy + y^2$ may not contain xy term, then $\theta =$ _____.
[0, $\frac{\pi}{4}$, $\frac{\pi}{2}$, π]

12. The conic $\frac{\ell}{r} = e\cos\theta$ represets a parabola if _____.
[$e = 0$, $e < 1$, $e = 1$, $e > 1$]

13. Two lines such that the pole of each line with respect to a conic lies on the other line are called _____.
[Conjugate lines, Polar lines, Transverse lines, Diameters]

14. The equation of pair of tangents from (x_1, y_1) to the conic $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ is given by _____.
[$S = T_1$, $S = T + T_1$, $SS_1 = T^2$, $SS_1 = TT_1$]

15. Direction cosines of a line normal to the plane $ax - by + cz = 10$ are proportional to _____.
[a, b, c; $a, -b, c$; $-a, -b, c$;; $a, b, -c$;]

16. If the equation $2x^2 - y^2 - \lambda z^2 + xy + 3yz = 0$ represents a pair of planes, then $\lambda =$ _____.
[-4, 0, 2, 4]

17. The equations to the straight lines through (a, b, c) parallel to the z axis are _____.
[$x = a, y = b$, $x = b, y = c$, $y = b, z = c$, $z = a, x = b$]

18. Let the equation of the circle be $S \equiv x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$, $U: \ell x + my + nz - 5 = 0$. The equation $S + kU = 0$, where k is any constant, represents _____.
[Circle , Pair of lines, Pair of Planes , Sphere]

19. The plane $\ell x + my + nz = p$ touches the sphere $x^2 + y^2 + z^2 = a^2$ if $p =$ _____.
[a , $\sqrt{\ell^2 + m^2 + n^2}$, $a\sqrt{\ell^2 + m^2 + n^2}$, $a^2\sqrt{\ell^2 + m^2 + n^2}$]

20. A triangle formed by three _____ of a sphere on its surface is called a spherical triangle.
[diameter of great circles, lines of great circle,
arcs of great circles, arcs of circles]

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SECTION "C"
[4 Q. × 7 = 28 marks]

1. What is the transformation of coordinate? Find the new equation of $5xy - 21x + 6y - 47 = 0$, when the axes are first translated to the point (1, 0) and rotated through the angle $\theta = 45^0$. [1+3+3]

OR

Find the axis and latus rectum of the parabola given by
$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$.

Prove that the lengths of semi-axes of the central conic $ax^2 + 2hxy + ay^2 = d$ are $\sqrt{\frac{d}{a+d}}$ and $\sqrt{\frac{d}{a-d}}$ respectively and their joint equation is $x^2 - y^2 = 0$. [4+3]

2. What is the line of the shortest distance? Find the length of the line of the shortest distance between the lines $\frac{x-3}{2} = \frac{y-4}{3} = \frac{z-5}{4}$ and $\frac{x-4}{3} = \frac{y-5}{4} = \frac{z-7}{5}$. Also, find the equation of the line of the shortest distance. [1+4+2]

3. Find the condition that the plane lx + my + nz = p should touch the sphere $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$. Also, find the equation of the sphere which touches the sphere $x^2 + y^2 + z^2 - x + 3y + 2z - 3 = 0$ at the point (1, 1, -1) and passes through origin. [3+4]

4. Define spherical radius and great circle. In spherical triangle ABC prove that :
$\cos B = \frac{\cos b - \cos a . \cos c}{\sin a . \sin c}$ [2+5]

SECTION "D"
[9 Q. × 3 = 27 marks]

5. Find the angle through which the axes must be rotated to remove the term containing xy in $x^2 + 2xy + 3y^2 - \sqrt{2}\,x = 0$, Find the transformed equation.

6. Find the equation of the normal to the conic $\frac{\ell}{r} = 1 + e\cos\theta$ at the point whose vectorial angle is θ.

OR

In any conic, prove that the tangents at the ends of any focal chord meet in the directrix.

7. Find the equations to the tangents to the conic $x^2 + 4xy + 3y^2 - 5x - 6y + 3 = 0$ which are parallel to the straight line $x + 4y = 0$.

8. Find the equation of the plane which contains the line $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z-3}{4}$ and perpendicular to the plane $x + 2y + z = 12$.

9. Find the polar of the point (2, 3) with respect to the conic
$$x^2 + 4xy - y^2 + 2x - 4y + 5 = 0$$

10. Determine the centre and radius of the sphere having the circle $x^2 + y^2 + z^2 = 9$, $x - 2y + 2z = 5$ as a great circle.

11. Prove that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ are coplanar. Find the equation of the plane in which they lie.

12. Transform the equations $x + y + z + 1 = 0$, $4x + y - 2z + 2 = 0$ of the line into symmetrical form.

13. Prove that the sines of the angles of a spherical triangle are proportional to the sines of the opposite sides.

OR

Prove that the sides of the angles of a polar triangle are respectively supplements of the sides and angles of the primitive triangle.