Marks scored:

KATHMANDU UNIVERSITY
End Semester Examination
February/March, 2019

Level : B. E.
Year : II

Course : MATH 205
Semester : I

Exam Roll No. :

Time: 30 mins.

F. M. : 20

Registration No.:

Date FEB 21 2019

SECTION "A"
[10Q. × 1= 10 marks]

Fill in the blank space (s) by most appropriate word (s) or symbol (s).

1. The origin of the coordinate axes should be transferred to $(\alpha, \beta) =$ ____________ so as to change the equation $x^2 + 2xy - y^2 + 2x - 2y + 1 = 0$ into an equation with linear terms missing.

2. When $e = 0$, $\frac{\ell}{r} = 1 + e\cos\theta$ where the symbols have their usual meanings, represents a(an) ________________________.

3. $ax^2 + 2hxy + by^2 = 1$ is the equation of ________________________________.

4. The length of a line whose projections on the axes are $1, 2, 2$ is ______________________.

5. Angle between the planes $x - 2y - z = 5$ and $2x - y + z = 2$ is ______________________.

6. The line $\frac{x}{2} = \frac{y}{3} = \frac{z}{4}$ meets the plane $x + y + z = 0$ at ______________________.

7. The equation $S_1 + \lambda S_2 = 0$ represents a(an) ____________________ through the intersection of the spheres $S_1 = 0$ and $S_2 = 0$ when $\lambda = -1$.

8. The equation of tangent plane to the sphere $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz = 0$ at $(0, 0, 0)$ is____________________________.

9. The sum of three sides of a spherical triangle is less than the ______________________ of a great circle.

10. The smallest arc joining two points on the surface of a sphere is the length of a _____________ circle.

SECTION "B"
[10 Q.× 1 = 10 marks]

Fill in the blank space (s) by choosing the most appropriate answer from among the given ones. **Do not tick** the answers.

11. The coordinate axes should be rotated through the angle ________ so that the expression $x^2 - 4xy + y^2$ may be reduced to the form $x'^2 + y'^2$.
[0; $\frac{\pi}{4}$; $\frac{\pi}{2}$; π]

12. The equation of chord of a conic $\frac{\ell}{r} = 1 + e\cos\theta$ joining the points whose vectorial angles are $\alpha + \beta$ and $\alpha - \beta$ is

[$\frac{\ell}{r} = \sec\frac{\beta-\alpha}{2}\cos\left(\theta - \frac{\beta+\alpha}{2}\right) + e\cos\theta$; $\frac{\ell}{r} = \sec\frac{\beta-\alpha}{2}\cos\left(\theta - \frac{\beta+\alpha}{2}\right) + e\cos\alpha$;
$\frac{\ell}{r} = \sec\beta\cos(\theta - \alpha) + e\cos\theta$; $\frac{\ell}{r} = \sec\frac{\beta-\alpha}{2}\cos(\theta - \alpha) + e\cos\theta$]

13. If the point (x_1, y_1) lies outside the conic then the polar is the same as ________________.

[Tangent, Chord, Chord of contact, Directrix]

14. The two tangents at (r_1, θ_1) and (r_2, θ_2) on a conic $\frac{\ell}{r} = 1 + e\cos\theta$ meet at (R, α) where $\alpha = \frac{\theta_2+\theta_1}{2}$ and $R =$ ________________.

[$\frac{\ell}{\cos\frac{\theta_2+\theta_1}{2} - e\cos\frac{\theta_2-\theta_1}{2}}$; $\frac{\ell}{\cos\frac{\theta_2+\theta_1}{2} + e\cos\frac{\theta_2-\theta_1}{2}}$;
$\frac{\ell}{\cos\frac{\theta_2-\theta_1}{2} - e\cos\frac{\theta_2+\theta_1}{2}}$; $\frac{\ell}{\cos\frac{\theta_2-\theta_1}{2} + e\cos\frac{\theta_2+\theta_1}{2}}$]

15. The ____________ of a conic section is a point such that all chords of the conic which pass through it are bisected there.

[Centre; Pole; Focus; Vertex]

16. The line $\frac{x-3}{2} = \frac{y-4}{3} = \frac{z-5}{4}$ and the plane $4x + 4y - 5z = 0$ are ________.

[Perpendicular; Parallel; Coincident; Non-coplanar]

17. The direction cosines of a line normal to the plane $2x + y - 2z = 0$ are ________________.

[2, 1, -2; $\frac{2}{\sqrt{3}}, \frac{1}{\sqrt{3}}, -\frac{2}{\sqrt{3}}$; $\frac{2}{3}, \frac{1}{3}, -\frac{2}{3}$; $\frac{1}{2}, 1, -\frac{1}{2}$]

18. When the distance between the centers of the sphere and the circle is not zero, then such circle is called________________ circle.

[Polar; Great; Small; Radical]

19. In any spherical triangle ABC, $\sin^2\frac{1}{2}A =$ ________________.

[$\frac{\sin(s)\sin(s-c)}{\sin(s-a)\sin b}$, $\frac{\sin(s)\sin(s-c)}{\sin a\sin b}$, $\frac{\sin(s-b)\sin(s-c)}{\sin b\sin c}$, $\frac{\sin(s)\sin(s-a)}{\sin b\sin c}$]

20. If A and B are the end points of a diameter and O is the centre of the sphere, then through A, B and O ____________ great circle(s) can be drawn.

[No, Exactly one, Finitely many, Infinitely many]

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SECTION "C"
[4 Q.× 7 = 28 marks]

1. Find the equation of the tangent at any point $P(x_1, y_1)$ of the conic section $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$. Show that the joint equation of the axes of the central conic $ax^2 + 2hxy + by^2 = 1$ is $h(x^2 - y^2) = (a - b)xy$. [4+3]

OR

What is chord of contact? Derive the equation of the chord of contact of the point $P(r', \alpha)$ with respect to the conic $\frac{\ell}{r} = 1 + e\cos\theta$. In any conic, prove that the tangents at the ends of any focal chord meet on the directrix. [1+3+3]

2. Define coplanar lines and skew lines. Find the shortest distance between the lines $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$ and $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$. Find the equation of the line of the shortest distance. [2+3+2]

3. Find the equation of the tangent planes to the sphere $x^2 + y^2 + z^2 + 2x - 4y + 6z - 7 = 0$ which intersect in a line $6x - 3y - 23 = 0 = 3z + 2$. Find the equation of the sphere having the circle $x^2 + y^2 + z^2 = 9,\ x - 2y + 2z = 5$ as a great circle. [4+3]

4. Define polar triangle. Prove that the sum of three sides of a spherical triangle is less than the circumference of a great circle and $\sin\frac{1}{2}A = \sqrt{\frac{\sin(s-b)\sin(s-c)}{\sin b \sin c}}$ where the symbols have their usual meanings. [1+3+3]

SECTION "D"
[9 Q.× 3 = 27 marks]

5. Prove that $g^2 + f^2$ is invariant of $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ under a rotation of axes without changing the origin.

OR

Translate the axes so as to change the equation $3x^2 - 2xy + 4y^2 + 8x - 10y + 8 = 0$ into an equation with first degree terms missing.

6. Find the equation of the chord of the conic $\frac{\ell}{r} = 1 + e\cos\theta$, joining the points whose vectorial angles are $\frac{\pi}{2}$ and $\frac{\pi}{6}$.

7. Identify the conic $9x^2 - 4xy + 6y^2 - 14x - 8y + 1 = 0$, find its centre.

8. Prove that $6x^2 + 4y^2 - 10z^2 + 3yz + 4zx - 11xy = 0$ represents a pair of perpendicular planes, find their equations.

9. Find the equation of the line through $(-2,3,4)$ and parallel to the planes $2x+3y+4z=5$ and $3x+4y+5z=6$.

10. Show that the lines $\frac{x+1}{-3}=\frac{y-3}{2}=\frac{z+2}{1}$ and $x=\frac{y-7}{-3}=\frac{z+7}{2}$ are coplanar, find the equation of the plane containing them.

11. A plane passes through a fixed point (a,b,c) and cuts the axes in A,B and C. Prove that the locus of the center of the sphere $OABC$ is $\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=2$.

12. Prove that the arc of a great circle drawn from a pole of great circle to any point in its circumference is a quadrant.

OR

Prove that the sides of the angles of a polar triangle are respectively supplements of the angles and sides of the primitive triangle.

13. Derive Sine rule for spherical triangle ABC.