KATHMANDU UNIVERSITY
End Semester Examination
February/March, 2018

MAR 0 4 2018

Level : B.E.
Year : II
Time : 2 hrs. 30 mins.

Course : MATH 205
Semester : I
F.M. : 55

SECTION "C"
[4 Q.× 7 = 28 marks]

1. Define conic section. Derive the equation of the chord joining two points $P(r_1, \theta_1)$ and $Q(r_2, \theta_2)$ on the conic $\frac{\ell}{r} = 1 + e\cos\theta$. In any conic, prove that the tangents at the ends of any focal chord meet on directrix. [1+3+3]

OR

Find the equation of the chord of the conic $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$, whose middle point is at $P(x_1, y_1)$. Show that the joint equation of the axes of the central conic $ax^2 + 2hxy + by^2 = 1$ is $h(x^2 - y^2) = (a - b)xy$. [4+3]

2. What are coplanar lines? Derive the condition that the lines $\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$ and $a_1x + b_1y + c_1z + d_1 = 0 = a_2x + b_2y + c_2z + d_2$ become coplanar. Find the equation of the plane containing the line $\frac{x-x_1}{\ell_1} = \frac{y-y_1}{m_1} = \frac{z-z_1}{n_1}$ and parallel to the line $\frac{x-x_2}{\ell_2} = \frac{y-y_2}{m_2} = \frac{z-z_2}{n_2}$. [1+3+3]

3. Find the equation of the tangent plane at a point $P(x_1, y_1)$ on the sphere $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$. A plane passes through a fixed point (a, b, c) and cuts the axes in A, B, C. Prove that the locus of the centre of the sphere $OABC$ is $\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 2$. [4+3]

4. Define spherical angle and spherical triangle. In any spherical triangle ABC, prove that $\cos A = \frac{\cos a - \cos b \cos c}{\sin b \sin c}$. [2+5]

SECTION "D"
[9 Q.× 3 = 27 marks]

5. By transforming to parallel axes through a properly chosen point (h, k), prove that the equation $12x^2 - 10xy + 2y^2 + 11x - 5y + 2 = 0$ can be reduced one containing only terms of second degree.

OR

What does the equation $(a - b)(x^2 + y^2) - 2abx = 0$ become if the origin be moved to the point $\left(\frac{ab}{a-b}, 0\right)$?

6. Prove that in any conic, the semi latus rectum is a harmonic mean between the segments of any focal chord.

7. Find the pole of the line $x + y + 9 = 0$ with respect to the conic $x^2 - 2xy + y^2 - 3x + y - 2 = 0$.

8. Find the equation of the plane through the points $(2, 2, 1)$ and $(9, 3, 6)$ and perpendicular to the plane $2x + 6y + 6z = 9$.

9. The plane $\ell x + my = 0$ is rotated about its line of intersection with the plane $z = 0$ through an angle α. Prove that the equation to the plane in its new position is $\ell x + my \pm (\sqrt{\ell^2 + m^2}) \tan \alpha \ z = 0$.

10. Find the magnitude of the line of the shortest distance between the lines $\frac{x}{4} = \frac{y+1}{3} = \frac{z-2}{2}$; $5x - 2y - 3z + 6 = 0 = x - 3y + 2z - 3$.

11. Find the equation of a sphere for which the circle $x^2 + y^2 + z^2 + 7y - 2z + 2 = 0, 2x + 3y + 4z = 8$ is a great circle.

12. Prove that the sum of the three sides of and spherical triangle is less than the circumference of a great circle.

OR

In any spherical triangle ABC, prove that $\sin \frac{1}{2} A = \sqrt{\frac{\sin(s-b)\sin(s-c)}{\sin b \sin c}}$, where the symbols have their usual meanings.

13. Prove that the sines of the angles of a spherical triangle are proportional to the sines of the opposite sides of the triangle.