KATHMANDU UNIVERSITY
End Semester Examination [C]
December, 2024

Marks Scored:

Level : B.E.
Year : II

Course : MATH 205
Semester : I

Exam Roll No. :

Time: 30 mins.

F. M. : 20

Registration No.:

Date : 20 DEC 2024

SECTION "A"
[10 Q. × 1 = 10 marks]

Fill in the blank space(s) by most appropriate words or symbol(s):

1. If the origin is transferred to $(-h,\ 0)$ and the axes are tuned through an angle θ, then the transformation will be given by the relations $x =$ _____, $y =$ _____ .

2. The equation of conic $\frac{l}{r} = 1 + e\cos\theta$ and _____represent the same conic.

3. The equation of directrix of $\frac{\ell}{r} = 1 + e\cos\theta$ is_____.

4. Two lines such that the pole of each with respect to conic passes satisfy the other are called _____lines.

5. If the line $\frac{x-3}{2} = \frac{y+2}{5} = \frac{z-7}{k}$ is parallel to the plane $2x - 4y + z = 3$, then $k =$ _____.

6. Line $\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$ and plane $ax + by + cz + d_2 = 0$ are perpendicular if _____.

7. The radius of the sphere $5x^2 + 5y^2 + 5z^2 + 2ux + 2vy + 2wz + d = 0$, is $r =$ _____.

8. If the radius of the circle is equal to the radius of the sphere, the circle is called _____.

9. The sum of three sides of a spherical triangle is_____.

10. The arc of a great circle drawn from a pole of great circle to any point in its circumference is a _____.

SECTION "B"

[10 Q. × 1 = 10 marks]

Fill in the blank space(s), ***DO NOT TICK,*** *by selecting the most appropriate answers from among the given ones.*

11. For the removal of the linear term from $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$, the rectangular axes must be shifted at the new origin without changing the direction axes is sat _____.

[$(\frac{hf-bg}{ab-h^2}, \frac{gh-af}{ab-h^2})$, $(\frac{hf-bg}{h^2-ab}, \frac{gh-af}{h^2-ab})$, $(\frac{hf+bg}{ab-h^2}, \frac{gh+af}{ab-h^2})$, $(\frac{gh-af}{ab-h^2}, \frac{hf-bg}{ab-h^2})$]

12. The conic $\frac{\ell}{r} = e\cos\theta$ represents a circle if _____.

[$e = 0$, $e < 1$, $e = 1$, $e > 1$]

13. If $\alpha_1, \alpha_2, \alpha_3$ be the angles that a line makes with the co-ordinate axes, then the value of $\cos^2\alpha_1 + \cos^2\alpha\_2 + \cos^2\alpha_3$ -2 will be _____.

[-1, 0, 1, -2]

14. The locus of the middle points of a system of parallel chords of a conic is called _____.

[conjugate axis, directrix, diameter, polar line]

15. Two planes represented by $ax^2 + by^2cz^2 + 2fyz + 2gzx + 2hxy = 0$ will be perpendicular to each other if

[$f + g + h = 0$, $a + b + c = 0$

$abc + 2fgh - af^2 - bg^2 - ch^2 = 0$, $a - b - c = 0$]

16. Direction cosines of a line normal to the plane $x + y + z = 3$ is _____.

[1, 1, 1, 3, 3, 3, $\frac{1}{3}, \frac{1}{3}, \frac{1}{3}$ $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$]

17. The equations to the straight lines through (a, b, c) parallel to the x-axis are _____.

[$x = a, y = b$, $x = b, y = c$, $y = b, z = c$, $z = a, x = b$]

18. If $S_1 = 0$ and $S_2 = 0$ are two spheres then $S_1 + \lambda S_2 = 0$ represents _____.

[Circle, Cylinder, Sphere, Straight Line]

19. The inclination of two arcs of great circles at their intersection points on a sphere's surface is called _____.

[*angular distance* spherical angle, *secondaries*, *spherical radius*]

20. All those great circles which pass through the poles of a given circle are _____ to the given circle.

[secondaries, reciporcal, primitives, polar]

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SECTION "C"
[4 Q. × 7 = 28 marks]

1. What is the directrix of contact? Find the equation of the directrix of the conic $\frac{\ell}{r} = 1 + e\cos\theta$. Show that $\frac{\ell}{r} = 1 + e\cos\theta$ and $\frac{\ell}{r} = -1 + e\cos\theta$ represent the same conic.

OR

Find the coordinates of the centre of $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$. Show that $x + 4y = 8$ is a tangent to the conic $x^2 + 4xy + 3y^2 - 5x - 6y + 3 = 0$.

2. Define the skew lines. Find the shortest distance between the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$. Also, find the equation of the line of the shortest distance. [1+4+2]

3. Derive the equation of the sphere when the ends of its diameter are given. Find the centre and radius of the circle in which the sphere $x^2 + y^2 + z^2 - 8x + 4y + 8z - 45 = 0$ is cut by the plane $x - 2y + 2z = 3$. [3+4]

4. Define spherical angle. Show that the sum of three sides of a spherical triangle is less than the circumference of a great circle. Show that: $\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c}$. [1+3+3]

SECTION "D"
[9 Q. × 3 = 27 marks]

5. Find the angle through which the axes must be rotated to remove the term containing xy in $ax^2 + 2hxy + by^2$.

OR

Prove that $g^2 + f^2$ is an invariant of $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ under a rotation of axes without changing the origin.

P.T.O.

6. Find the equation of the tangent at the point, whose vectorial angle is θ_1 of the conic $\frac{l}{r} = 1 + e\cos\theta$.

7. Define pole and polar in the conic section. Find the polar of (2, 3) concerning the conic $x^2 + 4xy - y^2 + 2x - 4y + 5 = 0$.

8. Show that the equation $2x^2 - 6y^2 - 12z^2 + 18yz + 2zx + xy = 0$ represents a pair of planes, find the angle between them.

9. Find the angle between the lines
$x - 2y + z = 0 = x + y - z$ and $x + 2y + z = 0 = 8x + 12y + 5z$.

10. Show that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ are coplanar. Find the equation of the plane in which they lie.

11. Find the equation of the sphere which has its centre at the origin and which touches the line $2(x + 1) = 2 - y = z + 3$.

12. Prove that the tangent planes to the spheres $x^2 + y^2 + z^2 + 2u + 2vy + 2wz + d = 0$ and $x^2 + y^2 + z^2 + 2u_1x + 2v_1y + 2w_1z + d_1 = 0$ at any common point are at right angles if $2uu_1 + 2vv_1 + 2ww_1 = d + d_1$.

13. Show that the sides of the angles of a polar triangle are respectively supplements of the angles and sides of the primitive triangle.

OR

The sum of the angles of a spherical triangle is greater than and less than 3π.