

KATHMANDU UNIVERSITY  
End Semester Examination [C]  
November/December, 2023

27 NOV 2023

Level : B.E.  
Year : II  
Time : 2 hrs. 30 mins.

Course : MATH 205  
Semester : I  
F. M. : 55

SECTION "C"

[4Q. × 7 = 28 marks]

1. Define the conic. Find the equation of asymptotes of the conic  $\frac{l}{r} = 1 + e \cos\theta$ . ( $e > 1$ ). Find the new equation of  $x^2 - 7xy + 5y^2 - 3x + 8y + 41 = 0$  when the axes are rotated through an angle  $\theta = \frac{\pi}{4}$ . [1+3+3]
2. What are skew lines and the line of shortest distance? Find the shortest distance between the lines  $\frac{x-3}{2} = \frac{y-4}{3} = \frac{z-5}{4}$  and  $\frac{x-4}{3} = \frac{y-5}{4} = \frac{z-7}{5}$ . Also, find the equation of the shortest distance. [2+3+2]

OR

Define coplanar lines. Find the condition that the line  $\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$  may lie on the plane  $ax + by + cz + d = 0$ . Find the equation of the plane through the line  $u = ax + by + cz + d = 0$  and  $v = a'x + b'y + c'z + d' = 0$  parallel to the line  $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$ . [1+3+3]

3. Define tangent plane. Find the equation of tangent plane at any point  $(\alpha, \beta, \gamma)$  of a sphere. Find the equation of the sphere which touches the sphere  $x^2 + y^2 + z^2 - x + 3y + 2z - 3 = 0$  at the point  $(1, 1, 1)$  and passes through the  $(0, 0, 0)$ . [1+3+3]
4. Define spherical angle and great circles? In spherical triangle ABC prove that :  
$$\cos A = \frac{\cos a - \cos b \cdot \cos c}{\sin b \cdot \sin c}$$
 [1+1+5]

SECTION "D"

[9 Q. × 3 = 27 marks]

5. What does the equation  $(x-a)^2 + (y-b)^2 = c^2$  become if the origin be moved to the point  $(a, b - c)$ ?
6. In any conic, prove that the tangents at ends of any focal chord meet on the directrix.
7. Define the tangent line of conic. Find the equation of tangent at  $(-2, 1)$  to the conic  $x^2 + 2xy - y^2 + 2x + 4y + 1 = 0$ .
8. Find the equation of the plane through the intersection of the planes  $x + 3y + 10z = 8$  and  $2x - 3y + 2z = 2$  and normal to the plane  $3x - 2y + 4z = 5$ .
9. Prove that the lines  $x = -2y + 7, z = 3y + 10$  and  $x = 5y - 1, z = 3y - 6$  are perpendicular to each other.

10. Prove that the lines  $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$  and  $4x - 3y + 1 = 0 = 5x - 3z + 2$  are coplanar. Also, find their point of intersection.
11. Find the equation of a sphere for which the circle  $x^2 + y^2 + z^2 + 7y - 2z + 2 = 0$ ,  $2x + 3y + 4z = 8$  is a great circle.
12. Find the equation of a tangent plane at the point (a, b, c) of the sphere  
$$x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$$
13. Prove that the sum of three sides of a spherical triangle is less than the circumference of a great circle.

**OR**

Prove that the sum of two sides of a spherical triangle is greater than the third side