KATHMANDU UNIVERSITY
End Semester Examination [C]
May/June, 2019

Marks Scored:

Level: B.Sc.
Year : II

Course : MATH 204
Semester : I

Exam Roll No. :

Time: 30 mins.

F. M. : 20

Registration No.:

Date 07 JUN 2019

SECTION "A"
[10 Q. × 1 = 10 marks]

Fill in the blank space (s) by most appropriate word (s) or symbol (s).

1. Changing the direction of axes without changing the origin is called _____ of axes.

2. The equation $\frac{\ell}{r} = e\cos\theta$ represents _____ of the conic $\frac{\ell}{r} = 1 + e\cos\theta$.

3. The locus of the middle points of a system of parallel chords of a conic is called _____.

4. If a line is a join of the two points $(1, 2, 3)$ and $(2, 3, 4)$, then its projections on the axes are _____.

5. The points $(1, -1, 2)$ and $(3, 2, -1)$ lie on _____ sides of the plane $x + 2y - 3z - 4 = 0$.

6. The lines which are not parallel and which do not intersect at a point are called _____.

7. Two planes $2x + 3y + kz = 5$ and $3x - 2y + 5z = 5$ are perpendicular, if $k =$ _____.

8. The equation of the tangent plane at (α, β, γ) of the sphere $x^2 + y^2 + z^2 = a^2$ is_____.

9. The tangent planes to the spheres $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$ and $x^2 + y^2 + z^2 + 2u_1x + 2v_1y + 2w_1z + d_1 = 0$ at any common point are at right angles if $2uu_1 + 2vv_1 + 2ww_1 =$_____.

10. Every homogeneous equation of second degree in x, y, z represents _____ with its vertex at origin.

SECTION "B"
[10 Q.× 1 = 10 marks]

Fill in the blank space (s) by choosing the most appropriate answer from among the given ones. **DO NOT TICK** the answers.

11. The equation $\frac{\ell}{r} = 1 + e\cos\theta$ represents a circle when $e =$ _____.

[0; $\frac{1}{2}$; 1; 2]

12. The polar of (r', α) with respect to the conic $\frac{\ell}{r} = 1 + e\cos\theta$ is _____, where the symbols have their usual meanings.

$[\left(\frac{\ell}{r} - e\cos\theta\right)\left(\frac{\ell}{r'} - e\cos\alpha\right) = \cos(\theta - \alpha);$
$\left(\frac{\ell}{r} - e\cos\theta\right)\left(\frac{\ell}{r'} - e\cos\theta\right) = \cos(\theta + \alpha);$
$\left(\frac{\ell}{r} - e\cos\theta\right)\left(\frac{\ell}{r'} - e\cos\theta\right) = \cos\alpha;$
$\left(\frac{\ell}{r} - e\cos\theta\right)\left(\frac{\ell}{r'} - e\cos\theta\right) = e\cos\alpha]$

13. The _____ of a conic section is a point such that all chords of the conic which pass through it are bisected there.
[Center; Focus; Vertex; Pole]

14. $by + cz + d = 0$ represents the plane parallel to _____.
[x −axis; y − axis; z −axis; $y = x$]

15. The line $\frac{x-3}{2} = \frac{y-4}{3} = \frac{z-5}{4}$ and the plane $4x + 4y - 5z = 0$ are _____.
[Perpendicular; Parallel; Coincident; Non-coplanar]

16. The direction cosines of a line normal to the plane $2x + y - 2z = 0$ are _____.
[2, 1, -2; $\frac{2}{\sqrt{3}}, \frac{1}{\sqrt{3}}, -\frac{2}{\sqrt{3}}$; $\frac{2}{3}, \frac{1}{3}, -\frac{2}{3}$; $\frac{1}{2}, 1, -\frac{1}{2}$]

17. When the distance between the centers of the sphere and the circle is not zero, then such circle is called_____circle.
[Polar; Great; Small; Radical]

18. Find the center of the sphere $2x^2 + 2y^2 + 2z^2 - 4x + 8y - 8z - 7 = 0$.
_____
[$(-4, 8, -8)$; $(-2, 4, -4)$;; $(-1, 2, -2)$; $(1, -2, 2)$]

19. Equation of enveloping cone of the sphere with the vertex at given point is _____.
[$S = T_1$; $SS_1 = T^2$; $S^2 = TT_1$; $SS_1 = TT_1$]

20. The locus of tangent line drawn from a given point to a given surface is known as _____ of that surface with the given point as vertex.
[Right circular cylinder; Enveloping cylinder;
Right circular cone; Enveloping cone]

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Time : 2 hrs. 30 mins.

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Semester : I
F.M. : 55

SECTION "C"
[3 Q. × 7 = 21 marks]

1. Define conic section. Derive the equation of directrix of the conic in polar form. PSP' and QSQ' are two perpendicular focal chords of a conic; prove that $\frac{1}{PP'}+\frac{1}{QQ'}$ is constant, where S is the pole of the conic. [1+3+3]

OR

Find the equation the chord of the conic section $ax^2+2hxy+by^2+2gx+2fy+c=0$, whose middle point is at $P(x_1,y_1)$. Find the middle point of the chord $9x-4y=14$ of the conic $2x^2+xy-3y^2=1$. [4+3]

2. What is the line of the shortest distance? Find the shortest distance between the lines $\frac{x-3}{1}=\frac{y-5}{-2}=\frac{z-7}{1}$ and $\frac{x+1}{7}=\frac{y+1}{-6}=\frac{z+1}{1}$. Find the equation of the line of the shortest distance. [1+3+3]

3. Find the center and radius of the circle in which the sphere $x^2+y^2+z^2-8x+4y+8z-45=0$ is cut by the plane $x-2y+2z=3$. Find the equations of the spheres which pass through the circle $x^2+y^2+z^2=5$, $x+2y+3z=3$ and touches the plane $4x+3y-15=0$. [3+4]

SECTION "D"
[6 Q. × 4 = 24 marks]

4. Find the equation of the chord of the conic $\frac{\ell}{r}=1+e\cos\theta$ joining the points whose vectorial angles are $\frac{\pi}{6}$ and $\frac{\pi}{2}$.

5. Find the pole of the line $x+y+9=0$ with respect to the conic
$$x^2-2xy+y^2-3x+y-2=0.$$

OR

Find the equation of pair of tangents from $(1,1)$ to the conic
$$2x^2+y^2-4x+2y+2=0.$$

6. Obtain the equation of the plane which passes through $(2,-3,1)$ and is perpendicular to the line joining the points $(3,4,-1)$ and $(2,-1,5)$.

7. Prove that the lines $\frac{x-1}{2}=\frac{y+1}{-3}=\frac{z+10}{8}$ *and* $\frac{x-4}{1}=\frac{y+3}{-4}=\frac{z+1}{7}$ are coplanar and find the plane on which they lie.

8. Find the equation of the sphere for which the circle $x^2+y^2+z^2+7y-2z+2=0$, $2x+3y+4z=8$ is a great circle.

9. Obtain the equation of the cylinder which passes through $y^2 = 4ax,\ z = 0$ and whose generators are parallel to the line $x = y = z$.

SECTION "E"

[5 Q. × 2 = 10 marks]

10. Find the equation of the curve $9x^2 + 4y^2 + 18x - 16y = 11$ referred to the parallel axes through $(-1, 2)$.

11. Obtain the equation of the plane through $(1, 0, 1)$ and parallel to the plane $x + 2y - z = 0$.

12. Where does the line $\frac{x-1}{2} = \frac{y+2}{-3} = \frac{z-3}{4}$ meet the plane $2x + 4y - z + 1 = 0$?

13. Find the equation of a sphere if one of its diameters has end points $(2, 1, 4)$ and $(4, 3, 10)$.

14. Prove that the cone $2x^2 + 2y^2 + 7z^2 - 10yz - 10zx + 2x + 2y + 26z - 17 = 0$ has the vertex at $(2, 2, 1)$.