SoSc -

KATHMANDU UNIVERSITY
End Semester Examination
March, 2025

Marks Scored:

Level : B.Sc.	Course : MATH 204
Year : II	Semester : I
Exam Roll No. :	Time: 30 mins.	F. M. : 20
Registration No.:	Date : 24 MAR 2025

SECTION "A"
[10 Q. × 1 = 10 marks]

Fill in the blank space(s) by writing the most appropriate word(s) or symbol(s).

1. If the origin is transferred to (α, β) and the axes are tuned through an angle θ, then the transformation will be given by the relations $x =$ ________________, $y =$ ________________.

2. The equation of the tangent to the conic $\frac{\ell}{r} = 1 + e\cos\theta$ at (r_1, θ_1) is $r =$ ______________.

3. The general equation of second degree, $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents an ellipse if $\Delta \neq 0$ and ________________, where the symbols have their usual meanings.

4. Direction cosines of $x -$ axis are ____________________________.

5. The angle between the pair of planes $2x + y + z = 2$ and $x - y - z = 5$ is ________________.

6. ________________ are the lines which are not parallel and which do not intersect at a point.

7. If the line $\frac{x-2}{2} = \frac{y+3}{5} = \frac{z-5}{p}$ and the plane $2x - 3y + z = 3$ are parallel, then $p =$ ________.

8. The radius of the sphere $x^2 + y^2 + z^2 - 2x + 4y - 6z - 2 = 0$ is ____________________.

9. The equation of the sphere described by the points $(0, 0, 0)$ and $(1, 1, 1)$ as the join of the diameter of the sphere is ________________________.

10. A surface generated by a straight line which passes through a fixed point and intersects a given curve is called a (an) ___________________.

SECTION "B"

[10 Q.× 1 = 10 marks]

Fill in the blank space(s), **DO NOT TICK**, by selecting the most appropriate answers from among the given ones.

11. The equation of the line, $x + y + 3 = 0$ referred to new parallel axes through $(-1, -2)$ becomes ________________.
[$x + y = 0$, $x + y - 3 = 0$, $x + y + 3 = 0$, $y - x + 3 = 0$]

12. If the two tangents at the points (r_1, θ_1) and (r_2, θ_2) on the conic $\frac{\ell}{r} = 1 + e\cos\theta$ intersect at the point (r', α), then $r' =$ ________________ and $\alpha = \frac{\theta_2+\theta_1}{2}$.
[$\frac{\ell}{\cos\frac{\theta_2-\theta_1}{2}+e\cos\frac{\theta_2+\theta_1}{2}}$, $\frac{\ell}{\cos\frac{\theta_2+\theta_1}{2}+e\cos\frac{\theta_2-\theta_1}{2}}$,
$\frac{\ell}{\cos\frac{\theta_2-\theta_1}{2}+r\cos\frac{\theta_2+\theta_1}{2}}$, $\frac{\ell}{r\cos\frac{\theta_2-\theta_1}{2}+\cos\frac{\theta_2+\theta_1}{2}}$]

13. What curve does the equation $x^2 + y^2 - 2xy - 2x - 1 = 0$ represent? ________________.
[circle ; parabola; ellipse; hyperbola]

14. The locus of the middle points of a system of parallel chords of a conic is called ________________.
[conjugate axis, directrix, diameter, polar]

15. The equation of the chord of contact of the tangents drawn from the point $(1, 1)$ on the conic $x^2 + 2xy - y^2 = 0$ is ________________.
[$x + 2y = 0$; $x - y = 0$; $x = 0$; $y = 0$]

16. $x = 0, y = 0$ represents ________________.
[x –axis; y – axis; z –axis; xy – plane]

17. Direction ratios of a line normal to the plane $2x - 3y + 5z = 7$ are ________________.
[$2, -3, 5$; $-3, 5, 7$; $5, 7, 2$; $\frac{2}{7}, -\frac{3}{7}, \frac{5}{7}$]

18. The straight line $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ intersects the plane $3x + 4y + 5z = 0$ at ________________.
[$(0, 0, 0)$; $(1, 2, 3)$; $(3, 4, 5)$; $(3, 8, 15)$]

19. The equation of the tangent plane at (α, β, γ) of the sphere $x^2 + y^2 + z^2 = a^2$ is ________________.
[$\alpha x + \beta y + \gamma z = 0$; $\alpha x + \beta y + \gamma z = \alpha^2 + \beta^2 + \gamma^2$;
$\alpha x + \beta y + \gamma z = a^2$; $\alpha x + \beta y + \gamma z = \alpha\beta\gamma$]]

20. If the generator of a cylinder is always at a constant distance from the fixed line, then the cylinder so generated is called ________________.
[principal cylinder; enveloping cylinder;
reciprocal cylinder; right circular cylinder]

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Semester : I
F. M. : 55

SECTION "C"

[3 Q. × 7 = 21 marks]

1. Define conic section. Derive the equation conic in a polar form whose focus is at the pole and the semi-latus rectum is ℓ. Prove that the locus of the pole of a chord which subtends a constant angle 2β at the focus of the conic $\frac{\ell}{r} = 1 + e\cos\theta$ is $\frac{\ell\sec\beta}{r} = 1 + e\,\sec\beta\,\cos\theta$. [1+3+3]

2. Prove that the lines $x = \frac{y-2}{2} = \frac{z+3}{3}$ and $\frac{x-2}{2} = \frac{y-6}{3} = \frac{z-3}{4}$ are coplanar, and find the plane through them. If the lines $x = ay + b, z = cy + d$ and $x = a'y + b', z = c'y + d'$ are perpendicular, show that $aa' + cc' + 1 = 0$. [4+3]

OR

Define skew lines and the line of the shortest distance. Find the shortest distance between the lines $\frac{x-3}{2} = \frac{y-4}{3} = \frac{z-5}{4}$ and $\frac{x-4}{3} = \frac{y-5}{4} = \frac{z-7}{5}$. Also, find the equation of the shortest distance. [2+3+2]

3. A sphere of radius, k passes through the origin and meets the coordinate axes at A, B, C. Prove that the centroid of the triangle ABC lies on the sphere, $9(x^2 + y^2 + z^2) = 4k^2$. Also, find the center and radius of the sphere having the circle $x^2 + y^2 + z^2 = 9$, $x - 2y + 2z = 5$ as a great circle. [3+4]

SECTION "D"

[6Q. × 4 = 24 marks]

4. What does the equation $x^2 + 2\sqrt{3}xy - y^2 = 2a^2$ become when the axes are turned through an angle 30^0 to the original axes?

5. Prove that the lengths of semi-axes of the central conic $ax^2 + 2hxy + ay^2 = d$ are $\sqrt{\frac{d}{a+d}}$ and $\sqrt{\frac{d}{a-d}}$ respectively and their joint equation is $x^2 - y^2 = 0$.

OR

Find the middle point of the chord, $9x - 4y = 14$ of the conic $2x^2 + xy - 3y^2 = 1$.

P.T.O.

6. Find the plane which passes through the point $(1, 2, -1)$ and contains the line $\frac{x+1}{2} = \frac{y-1}{3} = \frac{z+2}{-1}$.

7. Show that the equation to the line through (a, b, c) at right angles to the lines $\frac{x}{\ell_1} = \frac{y}{m_1} = \frac{z}{n_1}$ and $\frac{x}{\ell_2} = \frac{y}{m_2} = \frac{z}{n_2}$ is $\frac{x-a}{m_1n_2-m_2n_1} = \frac{y-b}{n_1\ell_2-n_2\ell_1} = \frac{z-c}{\ell_1m_2-\ell_2m_1}$.

8. Find the equations of the tangent planes to the sphere $x^2 + y^2 + z^2 + 2x - 4y + +6z - 7 = 0$ which intersects in the line $6x - 3y - 23 = 0 = 3z + 2$.

9. Find the equation of the cone with vertex at (α, β, γ) and the base $y^2 = 4ax,\ z = 0$.

SECTION "E"

[5Q. × 2 = 10 marks]

10. Transform to parallel axes through the point $(-1, 2)$ the equation $x^2 - y^2 + 2x + 4y = 0$.

11. If PSQ is a focal chord of the conic $\frac{\ell}{r} = 1 + e\cos\theta$, show that $\frac{1}{SP} + \frac{1}{SQ} = \frac{2}{\ell}$.

12. A straight line passes through $(1, 1, 1)$ and is parallel to the plane $2x + 3y + 4z = 5$. Find the equation of the line.

13. Find the intercepts made on the coordinate axes by the plane $x + 2y - 5z = 0$.

14. Determine the equation of the sphere that passes through the points $(0, 0, 0)$, $(1, 0, 0)$, $(0, 1, 0)$, and $(0, 0, 1)$.