

KATHMANDU UNIVERSITY
End Semester Examination
March/April, 2017

Marks Scored:

Level : B. Sc.

Year : II

Exam Roll No. :

Time : 30 mins.

Course : MATH 204

Semester : I

F. M. : 20

Registration No. :

Date : APR 10 2017

SECTION "A"

[10 Q. × 1 = 10 marks]

Fill in the blank space (s) by most appropriate word (s) or symbol (s).

1. The equation of the curve $y = x^2$ referred to new parallel axes through $(-1, 2)$ becomes _____.
2. The tangent at the point whose vectorial angle is θ_1 of the conic $\frac{\rho}{r} = 1 + e \cos\theta$ is _____.
3. The locus of the middle points of a system of parallel chords of a conic is called _____.
4. If α, β, γ be the angles which a line makes with the co-ordinate axes, then $\sin^2\alpha + \sin^2\beta + \sin^2\gamma - 2 =$ _____.
5. Angle between the pair of planes $x - 3y - z = 0$ and $x - y + 4z = 0$ is _____.
6. If the line $\frac{x-2}{2} = \frac{y+3}{5} = \frac{z-5}{k}$ is parallel to the plane $2x - 3y + z = 3$, then $k =$ _____.
7. Lines which are not parallel and which do not intersect at a point are called _____.
8. Any equation of the type $cx^2 + cy^2 + cz^2 + 2ux + 2vy + 2wz + d = 0, c \neq 0$ always represents a sphere with centre at _____.
9. The equation of the sphere described on the joint of $(0, 0, 0)$ and $(1, 2, 3)$ as diameter is _____.
10. Every homogenous equation of second degree in x, y, z represents a (an) _____ whose vertex is at origin.

SECTION "B"

[10 Q. × 1 = 10 marks]

Fill in the blank space (s) by choosing the most appropriate answer from among the given ones.
Do not tick the answers.

11. If the axes be rotated through an angle θ so that the expression $x^2 + xy + y^2$ may not contain xy term, then $\theta =$ _____.
[0, $\frac{\pi}{4}$, $\frac{\pi}{2}$, π]

12. The conic $\frac{\rho}{r} = e \cos\theta$ represents an ellipse if _____.
 [$e = 0$, $e < 1$, $e = 1$, $e > 1$]
13. Two lines such that the pole of each line with respect to a conic lies on the other line are called _____.
 [Conjugate lines, Polar lines, Transverse lines, Diameters]
14. The equation of pair of tangents from (x_1, y_1) to the conic $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ is given by _____.
 [$S = T_1$, $S = T + T_1$, $SS_1 = T^2$, $SS_1 = TT_1$]
15. Direction cosines of a line normal to the plane $2x - y + 3z = 10$ are proportional to
 [$2, -1, 3$; $-1, 3, 2$; $-1, 3, 10$; $10, 2, -1$]
16. If the equation $2x^2 - y^2 - \lambda z^2 + xy + 3yz = 0$ represents a pair of planes, then $\lambda =$ _____.
 [-4 , 0 , 2 , 4]
17. The equations to the straight lines through (a, b, c) parallel to the x axis are _____.
 [$x = a, y = b$, $x = b, y = c$, $y = b, z = c$, $z = a, x = b$]
18. Let the equation of the circle be $S \equiv x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$, $U: \ell x + my + nz - p = 0$. The equation $S + \lambda U = 0$, where λ is any constant, represents _____.
 [Circle, Pair of lines, Pair of Planes, Sphere]
19. Three planes intersect in a point if _____ where $\Delta_1, \Delta_2, \Delta_3, \Delta_4$ have their usual meanings.
 [$\Delta_1 = \Delta_2 = \Delta_3$, $\Delta_4 \neq 0$, $\Delta_4 = 0$ but $\Delta_3 \neq 0$, $\Delta_1 = \Delta_3 = 0$]
20. If the generator of a cylinder is always at a constant distance from the fixed line, then the cylinder so generated is called a (an) _____ cylinder.
 [Right circular, Enveloping, Oblique, Parabolic]

KATHMANDU UNIVERSITY
End Semester Examination
March/April, 2017

APR 10 2017

Level : B.Sc.
Year : II
Time : 2 hrs. 30 mins.

Course : MATH 204
Semester : I
F.M. : 55

SECTION "C"

[3 Q. × 7 = 21 marks]

1. What is a conic section? Derive the polar equation of a conic section. Prove that the locus of the pole of a chord which subtends a constant angle 2β at the focus of the conic $\frac{\ell}{r} = 1 + e \cos\theta$ is $\frac{\ell \sec\beta}{r} = 1 + e \sec\beta \cos\theta$. [1+3+3]
 2. Define a line of shortest distance. Find the shortest distance between the lines $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$ and $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$. Find the equation of the line of the shortest distance. [1+4+2]
- OR**
- How is a straight line defined in 3D? Derive the equation of a line through a given point and given direction in the form $\frac{x-x_1}{\ell} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$. Find the distance of the point $(-1, -5, -10)$ from the point of intersection of the line $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12}$ and the plane $x - y - z = 5$. [1+3+3]
3. Derive the equation of a tangent plane at any point (α, β, γ) of the sphere $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$. A plane passes through a fixed point (a, b, c) and cuts the axes in A, B, C . Prove that the locus of the centre of the sphere $OABC$ is $\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 2$. [1+3+3]

SECTION "D"

[6 Q. × 4 = 24 marks]

4. By transforming to parallel axes through properly chosen point (h, k) , prove that the equation $12x^2 - 10xy + 2y^2 + 11x - 5y + 2 = 0$ can be reduced to one containing only terms of second degree.
5. If the tangent at any point P of a conic meets the directrix in K , prove that KSP is a right angle.
6. Show that the conic $x^2 - 4xy + y^2 - 2x - 20y - 11 = 0$ is hyperbola. Also, show that $(-7, -4)$ is its centre.

OR

Prove that the lengths of the semi-axes of the conic $ax^2 + 2hxy + ay^2 = d$ are $\sqrt{\frac{d}{a+h}}$ and $\sqrt{\frac{d}{a-h}}$ respectively and that their equations are $x^2 - y^2 = 0$.

7. Prove that the equation $6x^2 + 4y^2 - 10z^2 + 3yz + 4zx - 11xy = 0$ represents a pair of perpendicular planes, find their separate equations.
8. Find the centre and radius of the circle in which the sphere $x^2 + y^2 + z^2 - 8x + 4y + 8z - 45 = 0$ is cut by the plane $x - 2y + 2z = 3$.
9. Show that the equation to the cone whose vertex is (α, β, γ) and the base parabola $z^2 = 4ax, y = 0$ is $(\beta z - \gamma y)^2 = 4a(\beta - \gamma)(\beta x - \alpha y)$.

SECTION "E"

[5 Q. × 2 = 10 marks]

10. What does the equation $(x - a)^2 + (y - b)^2 = c^2$ become when it is transferred to parallel axes through the point $(a - c, b)$?
11. Obtain the equation of the plane through the intersection of the planes $x + 2y + 3z + 4 = 0$, $4x + 3y + 2z + 1 = 0$ and the origin.
12. Prove that the lines $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ and $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ are coplanar.
13. Find the equation of the sphere having the circle, $x^2 + y^2 + z^2 = 9$, $x - 2y + 2z = 5$ as a great circle.
14. Find the vertex of the cone $x^2 - 2y^2 + 3z^2 + 5yz - 6zx - 4xy + 8x - 19y - 2z - 20 = 0$.