KATHMANDU UNIVERSITY
End Semester Examination
July/August, 2024

Level : B.Sc.
Year : II
Time : 2 hrs. 30mins.

20 AUG 2024

Course : MATH 204
Semester : I
F. M. : 55

SECTION "C"
[3 Q. × 7 = 21 marks]

1. Derive the equation of the conic section in polar form. State when the equation represents ellipse, hyperbola, parabola, and circle. If a chord PQ of a conic whose eccentricity is e, and the semi-latus rectum is ℓ subtends a right angle at the focus S, prove that $\left(\frac{1}{SP}-\frac{1}{\ell}\right)^2+\left(\frac{1}{SQ}-\frac{1}{\ell}\right)^2=\left(\frac{e}{\ell}\right)^2$. [2+2+3=7]

2. What is plane? Derive the equation of the plane in intercept form. Find the equation of the plane through the line $u = ax + by + cz + d = 0$, $v = \alpha x + \beta y + \gamma z + \delta = 0$ and parallel to the line $\frac{x}{l}=\frac{y}{m}=\frac{z}{n}$. [1+3+3=7]

OR

Derive the condition for the lines $\frac{x-x_1}{\ell_1}=\frac{y-y_1}{m_1}=\frac{z-z_1}{n_1}$ and $\frac{x-x_2}{\ell_2}=\frac{y-y_2}{m_2}=\frac{z-z_2}{n_2}$ to be coplanar. Show that the lines $\frac{x-3}{2}=\frac{y-4}{3}=\frac{z-5}{4}$ and $\frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5}$ are coplanar. Find the equation of the plane in which they lie. [3+3+1=7]

3. Define the concepts of small and great circles of a sphere. If a circle of radius of k passes through the origin and meets the axes in A, B, and C, show that the centroid of the triangle ABC lies on the sphere $9(x^2+y^2+z^2)=4k^2$. Also, obtain the equation of the sphere through the circle $x^2+y^2+z^2=5,\ x+2y+3z=3$ which touches the plane $4x+3y=15$. [2+3+2=7]

SECTION "D"
[6 Q. × 4 = 24 marks]

4. If the origin is moved to the point $\left(\frac{mn}{m-n}, 0\right)$, what does the equation $(m-n)(x^2+y^2)-2mn\,x=0$ become?

5. Prove that the lengths of the semi-axes of the central conic $ax^2+2hxy+ay^2=d$ are $\sqrt{\frac{d}{a+h}}$ and $\sqrt{\frac{d}{a-h}}$ respectively and their joint equation is $x^2-y^2=0$.

OR

Find the pole of the line $x+y+9=0$ with respect to the conic $x^2-2xy+y^2-3x+y-2=0$.

P.T.O.

6. If the tangent at any point P of the conic $\frac{\ell}{r} = 1 + e\cos\theta$ meets the directrix at K, prove that KSP is a right angle.

7. Find the shortest distance between the lines $\frac{x}{2} = \frac{y}{-3} = \frac{z}{1}$ and $\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z+2}{2}$. Also, find the equation of the line of the shortest distance.

8. Obtain the equations to the sphere having the circle $x^2 + y^2 + z^2 = 9,\ x - 2y + 2z = 5$ as a great circle, and determine its center and radius.

9. Derive the equation of the cone with vertex at (α, β, γ) and base $y^2 = 4ax,\ z = 0$.

SECTION "E"

[5 Q. × 2 = 10 marks]

10. If the rectangular coordinate axes are turned through the angle $\frac{\pi}{4}$, what does the equation $x^2 + y^2 = 4$ become?

11. Show that the line $x + 4y = 8$ touches the conic $x^2 + 4xy + 3y^2 - 5x - 6y + 3 = 0$.

12. Where does the line $\frac{x-1}{2} = \frac{y+2}{-3} = \frac{z+3}{4}$ meet the plane $2x + 4y - z + 1 = 0$?

13. A plane meets the axes of coordinates at P, Q, and R. If (α, β, γ) is the centroid of the triangle PQR, find the equation of the plane.

14. Find the equation of the sphere with center at $(1, -2, 3,)$ and radius 3.

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SECTION "A"
[10 Q. × 1 = 10 marks]

Fill in the blank space(s) by most appropriate words or symbol(s):

1. If the origin is transferred to (α, β) and the axes are turned through an angle θ, the transformation will be given by the relations $x =$ _____, $y =$ _____.

2. Equation of the tangent to the conic $\frac{\ell}{r} = 1 + e\cos\theta$ at a point whose vectorial angle is θ_1 is given by _____.

3. If the point (x_1, y_1) is outside the conic, then the polar is the same as _____ of the point.

4. The equation of the chord of contact of tangents from the point (x_1, y_1) to the conic $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ is _____.

5. The direction cosines of the $x-$ axis are _____.

6. The angle between the planes $x + 4y + 6z = 0$ and $2x + y - z = 0$ is _____.

7. Lines which are not intersecting and _____ are known as skew lines.

8. The center of the sphere $x^2 + y^2 + z^2 - 2x - 4y - 6z + 1 = 0$ is at _____.

9. If the radius of the circle is equal to the radius of the sphere, the circle is called _____.

10. The surface generated by a variable straight line which is parallel to a fixed line (satisfying one more condition) is called _____.

SECTION "B"

[10 Q. × 1 = 10 marks]

Fill in the blank space(s), ***DO NOT TICK,*** *by selecting the most appropriate answers from among the given ones.*

11. When the axes are turned through the angle α, the equation $x\cos\alpha + y\sin\alpha = p$ of a line becomes _____.

[$x = p$ $y = p$ $x + y = p$ $xy = p$]

12. The locus of the middle points of a system of parallel chords of a conic is called _____.

[chord diameter pole conjugate]

13. $\frac{\ell}{r} = e\cos\theta$ is the equation of _____ of the conic $\frac{\ell}{r} = 1 + e\cos\theta$.

[polar directrix tangent chord]

14. The equation $x^2 + y^2 - 2xy - 2x - 1 = 0$ represents _____.

[hyperbola ellipse parabola circle]

15. The projection of a line joining two points $(1,2,3)$ and $(2,3,4)$ on the axes are _____.

[$1,2,3$ $2,3,4$ $1,1,1$ $2,6,12$]

16. The intercepts made on the coordinate axes by the plane $x + 2y - 2z = 4$ are _____.

[$1,2,-2$ $\frac{1}{4},\frac{1}{2},-\frac{1}{2}$ $4,2,-2$ $4,16,-16$]

17. If the lines $\frac{x-1}{-3} = \frac{y-2}{k} = \frac{z-3}{2}$ and $\frac{x-1}{3k} = \frac{y-5}{5} = \frac{z-6}{2}$ are perpendicular to each other, then $k =$ _____.

[0 -1 1 2]

18. Equation of the tangent to the sphere $x^2 + y^2 + z^2 = 9$ at $(1,-2,2)$ is _____.

[$x - 2y + 2z = 9$ $x + 2y - 2z = 9$ $x + y + z = 9$ $x + 4y + 4z = 9$]

19. The equation of the enveloping cone of the sphere $x^2 + y^2 + z^2 = a^2$ with vertex at (α,β,γ) is given by… … … … … …, where S, S_1, T have their usual meanings.

[$SS_1 = T^2$ $S_1T = S^2$ $ST = S_1$ $T = S_1 + S$]

20. Every homogeneous equation of the second order in x, y, z represents a _____.

[cone cylinder paraboloid ellipsoid]