KATHMANDU UNIVERSITY
End Semester Examination
June/July, 2023

Marks Scored:

Level : B.Sc.
Year : II

Course : MATH 204
Semester : I

Exam Roll No. :

Time: 30 mins.

F. M. : 20

Registration No.:

Date : 18 JUL 2023

SECTION "A"
[10Q. × 1 = 10 marks]

Fill in the blank space(s) by writing most appropriate word(s) or symbol(s).

1. The equation of the curve $y = x^2$ referred to new parallel axes through (-1, 2) becomes _____.

2. The equation of the directrix of the conic $\frac{l}{r} = 1 + e\cos\theta$ is given by _____.

3. The general equation of second degree $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents an ellipse if $\Delta \neq 0$ and _____, where symbol has usual meaning.

4. Two points such that the polar of each with respect to a conic pass through the other are called_____ points.

5. The locus of middle points of a system of parallel chords of a conic is called _____ of the conic.

6. If the planes $2x + 3y + 4z = 7$ and $2x - my + 5z = 9$ are parallel, then $m =$ _____.

7. If the line $\frac{x-2}{2} = \frac{y+3}{5} = \frac{z-5}{k}$ is parallel to the plane $2x - 3y + z = 3$, then $k =$ _____.

8. Centre of the sphere $mx^2 + my^2 + mz^2 + 2ax + 2by + 2cz + d = 0,\ m \neq 0$ is _____.

9. The equation of the tangent plane at (α, β, γ) of the sphere $x^2 + y^2 + z^2 = r^2$ is _____.

10. The locus of the tangents to a given surface in a given direction is called the _____ cylinder of the surface.

SECTION "B"

[10Q.× 1 = 10 marks]

Fill in the blank space (s) by choosing the most appropriate answers from among the given ones. **Do not tick** the answers.

11. The angle through which the axes should be turned so that the equation $x^2 + xy + y^2 = 0$ may be reduced to the equation without xy term is _____.
[0^0; $\frac{\pi}{6}$; $\frac{\pi}{4}$; $\frac{\pi}{2}$]

12. The polar of the point (r', α) with respect to the conic $\frac{\ell}{r} = 1 + e\cos\theta$ is _____.
[$\frac{\ell}{r} = \cos(\theta - \alpha) + e\cos\theta$; $\left(\frac{\ell}{r'} - \cos(\theta - \alpha)\right)\left(\frac{\ell}{r} - \cos\theta\right) = e\cos\theta$;
$\left(\frac{\ell}{r'} - \cos\alpha\right)\left(\frac{\ell}{r} - \cos\theta\right) = e\cos\theta$; $\left(\frac{\ell}{r'} - e\cos\alpha\right)\left(\frac{\ell}{r} - e\cos\theta\right) = cos(\theta - \alpha)$]

13. The equation of chord of a conic $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ whose middle point is at $P(x_1, y_1)$ is given by _____.
[$T = S$; $T = S_1$; $T_1 = S$; $T_1 = S_1$]

14. The center of the conic $x^2 - 4xy + y^2 - 2x - 20y - 11 = 0$ _____.
[$(7, -4)$; $(-7, 4)$; $(-7, -4)$; $(7, 4)$]

15. For what the value of k the equation $2x^2 - y^2 - kz^2 + xy + 3yz = 0$ represents a pair of perpendicular planes? _____.
[-2; -1; 1; 2]

16. Direction cosines of a line normal to the plane $x - y + z = 7$ are proportional to _____.
[1, -1, 1; 1, -1, -2; $\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$; $\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}$]

17. The two lines $\frac{x+1}{m} = \frac{y-3}{2} = \frac{z+2}{1}$ and $\frac{x}{1} = \frac{y-7}{-3} = \frac{z+7}{2}$ are coplanar if $m =$_____.
[-1; 1; -3; 3]

18. Skew lines are _____.
[intersecting; coincident; parallel; non-coplanar]

19. If $S_1 = 0$ and $S_2 = 0$ are two spheres then $S_1 + \lambda S_2 = 0$ represents _____.
[circle; cylinder; sphere; straight line]

20. Condition that the cone $ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy = 0$ has three mutually perpendicular generators is _____.
[$a + b + c = 0$; $a - b - c = 0$; $a - b + c = 0$; $abc = 0$]

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Semester : I
F.M. : 55

SECTION "C"
[3Q. × 7 = 21 marks]

1. What is a conic section? Derive the equation of a conic section. If a chord PQ of conic whose eccentricity is e and semi-latus rectum is l subtends a right angle at the focus S, then prove that $\left(\frac{1}{SP}-\frac{1}{l}\right)^2+\left(\frac{1}{SQ}-\frac{1}{l}\right)^2=\frac{e^2}{l^2}$. [1+3+3]

2. How is the angle between a plane and a straight line defined? Derive the condition for the general equation of second degree to represent a pair of planes. Find the equation of the plane containing the line $2x-5y+2z-6=0, 2x+3y-z-5=0$ and parallel to the line $x=-\frac{y}{6}=\frac{z}{7}$. [1+3+3]

OR

Define a straight line in three dimension. Derive the condition for a line to lie on a plane. Find the length and the equation of the shortest distance between the lines
$3x-9y+5z=0=x+y-z$ and $6x+8y+3z-13=0=x+2y+z-3$. [1+3+3]

3. Define a sphere. Derive the equation of sphere when two ends of its diameter are given. Show that the plane $2x-y+2z=14$ touches the sphere
$x^2+y^2+z^2-4x+2y-4=0$. Also, find the point of contact. [1+3+3]

SECTION "D"
[6 Q.× 4 = 24 marks]

4. By a suitable rotation, if $ux+vy$ becomes $UX+VY$, where u and v are independent of x and y; prove that $u^2+v^2=U^2+V^2$.

5. What is focal chord? If PSP' and QSQ' be any two focal chords of a conic which are at right angles to one another, prove that $\frac{1}{SP.SP'}+\frac{1}{SQ.SQ'}$ is constant.

6. What conic does the equation $14x^2-4xy+11y^2-44x-58y+71=0$ represent? Find the centre of the given conic.

OR

Define conjugate points and then show that the points (1, 2) and (-2, 3) are conjugate with respect to the conic $2x^2+6xy+y^2+4x-2y+8=0$.

7. Prove that the lines $x = ay + b,\ z = cy + d$ and $x = a'y + b',\ z = c'y + d'$ are perpendicular if $aa' + cc' + 1 = 0$.

8. Prove that the tangent planes to the spheres $x^2 + y^2 + z^2 + 2u + 2vy + 2wz + d = 0$ and $x^2 + y^2 + z^2 + 2u_1x + 2v_1y + 2w_1z + d_1 = 0$ at any common point are at right angles if $2uu_1 + 2vv_1 + 2ww_1 = d + d_1$.

9. Define the reciprocal cone and right circular cylinder. Find the equation of the right circular cylinder of radius 4 and axis the line $x = 2y = -z$.

SECTION "E"

[5Q. × 2 = 10 marks]

10. Determine the new equation of $3x^3 - 18x^2 + 36x - 4y - 36 = 0$ if the origin is translated to the given point $(2, -3)$.

11. Find the angle between the planes $x - 2y - z = 5$, and $2x - y + z = 2$.

12. Find where the line $\frac{x-1}{2} = \frac{y+2}{-3} = \frac{z+3}{4}$ meet the plane $2x + 4y - z + 1 = 0$.

13. Find the equation of the sphere having the circle $x^2 + y^2 + z^2 = 9,\ x - 2y + 2z = 5$ as a great circle.

14. Prove that the cone $ayz + bzx + cxy = 0$ and $\sqrt{ax} + \sqrt{by} + \sqrt{cz} = 0$ are reciprocal.