Marks scored:

KATHMANDU UNIVERSITY
End Semester Examination [C]
July, 2017

Level : B. Sc. | Course : MATH 204
Year : II | Semester: I
Exam Roll No. : | Time: 30 mins. | F. M. : 20
Registration No.: | Date JUL 1 3 2017

SECTION "A"
[10Q. × 1 = 10 marks]

Fill in the blank space (s) by most appropriate word (s) or symbol (s).

1. To change the origin of a system of coordinate axes without changing the directions of the axes is called _____ of axes.

2. The equation of the chord of the conic $\frac{\ell}{r} = 1 + e\cos\theta$ joining the points whose vectorial angles are $(\alpha - \beta), (\alpha + \beta)$ is _____.

3. Two lines such that the pole of each line with respect to a conic lies on the other line are called _____.

4. If a, b, c be the angles which a line makes with the co-ordinate axes, then $\cos^2 a + \cos^2 b + \cos^2 c =$ _____.

5. The length of perpendicular from a point $(0, 0, 0)$ to the plane $x - 2y + 2z + 3 = 0$ is _____.

6. If the line $\frac{x-2}{2} = \frac{y+3}{5} = \frac{z-5}{k}$ is parallel to the plane $2x - 3y + z = 3$, then $k =$ _____.

7. The perpendicular distance between two non-intersecting lines (non-parallel) lines is known as the line of _____.

8. $ax^2 + ay^2 + az^2 + 2ux + 2vy + 2wz + d = 0$, $a \neq 0$ is a sphere with radius, $R=$ _____.

9. If $S_1 = 0$ and $S_2 = 0$ are two spheres then, $S_1 - S_2 = 0$ represents _____.

10. Every homogenous equation of second degree in x, y, z represents a (an) _____ whose vertex is at origin.

SECTION "B"
[10 Q.× 1 = 10 marks]

Fill in the blank space (s) by choosing the most appropriate answer from among the given ones. **Do not tick** the answers.

11. If the axes be rotated through an angle θ so that the expression $x^2 + xy + y^2$ may not contain xy term, then $\theta =$ _____.

[0, $\frac{\pi}{4}$, $\frac{\pi}{2}$, π]

12. If the axis of the conic makes an angle α with the initial line, the equation of the conic becomes _____.
$\left[\frac{\ell}{r} = 1 + e\cos\alpha,\ \ \frac{\ell}{r} = 1 - e\cos\alpha,\ \ \frac{\ell}{r} = 1 + e\cos(\theta + \alpha),\ \ \frac{\ell}{r} = 1 + e\cos(\theta - \alpha)\right]$

13. The locus of the middle points of a system of parallel chords of a conic is called _____.
[Diameter, Polar line, Transverse line, Tangent]

14. The equation of pair of tangents from (x_1, y_1) to the conic $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ is given by _____.
[$S = T_1$, $S = T + T_1$, $SS_1 = T^2$, $SS_1 = TT_1$]

15. Angle between the pair of planes $x + 2y - 3z = 4$ and $7x + y + 3z = 5$ is _____.
[0; $\frac{\pi}{4}$; $\frac{\pi}{2}$; π]

16. The two planes $ax + by + cz + d = 0$ and $\alpha x + \beta y + \gamma z + \delta = 0$ will be parallel if _____.
[$a\alpha + b\beta + c\gamma = 0$, $\frac{a}{\alpha} = \frac{b}{\beta} = \frac{c}{\gamma}$, $a = b\beta + c$, $\alpha = b\beta + c\gamma$]

17. The two planes represented by $ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy = 0$ will be perpendicular to each other if _____.
[$a + b - c = 0$, $a + b + c = 0$, $f^2 = ah + bg$, $f^2 + g^2 + h^2 = ab + bc + ca$]

18. $ax^2 + by^2 + cz^2 + 2ux + 2vy + 2wz + d = 0$ represents a sphere if _____.
[$a = b = c$, $a = b - c$, $a = b + c$, $a = bc$]

19. Three planes intersect in a point if _____ where Δ_1, Δ_2, Δ_3, Δ_4 have their usual meanings.
[$\Delta_1 = \Delta_2 = \Delta_3$, $\Delta_4 \neq 0$, $\Delta_4 = 0$ but $\Delta_3 \neq 0$, $\Delta_1 = \Delta_3 = 0$]

20. A surface generated by a variable straight line which is parallel to fixed line is a _____.
[Cylinder, Cone, Circle, Guiding Curve]

KATHMANDU UNIVERSITY
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Level : B.Sc.
Year : II
Time : 2 hrs. 30 mins.

Course : MATH 204
Semester : I
F.M. : 55

SECTION "C"
[3 Q.× 7 = 21 marks]

1. Define conic section. Derive the equation of a directrix of the conic $\frac{\ell}{r} = 1 + e\cos\theta$. If PSP' and QSQ' be any two focal chords of a conic which are at right angles to one another; prove that $\frac{1}{SP.SP'} + \frac{1}{SQ.SQ'}$ is constant. [1+3+3]

2. What is a line of shortest distance? Find the shortest distance between the lines $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$ and $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$. Find the equation of the line of the shortest distance. [1+3+3]

OR

Define plane. Find the equation of the plane in the normal form. Find the intercepts made on the coordinate axes by the plane $x + 2y - 2z = 9$ and find the direction cosines of the line normal to this plane. [1+3+3]

3. Define great circle and small circle. Derive the equation of a sphere when the ends of its diameter are given. Find the centre and radius of the circle in which the sphere $x^2 + y^2 + z^2 - 8x + 4y + 8z - 45 = 0$ is cut by the plane $x - 2y + 2z = 3$. [1+3+3]

SECTION "D"
[6 Q.× 4 = 24 marks]

4. Find the angle through which the axes must be rotated to remove the term containing xy in $3x^2 + 2xy + 3y^2 - \sqrt{2}\,x = 0$. Also, find the transformed equation.

5. Prove that in any conic, the semi-latus rectum is a harmonic mean between the segments of any focal chord.

6. Find the middle point of the chord, $9x - 4y = 14$ of the conic $2x^2 + xy - 3y^2 = 1$.

OR

Prove that the lengths of the semi-axes of the conic $ax^2 + 2hxy + ay^2 = d$ are $\sqrt{\frac{d}{a+h}}$ and $\sqrt{\frac{d}{a-h}}$ respectively and that their equations are $x^2 - y^2 = 0$.

7. Find the equation of the plane through the plane of intersection of the planes $2x + 3y + 10z = 8$, $2x - 3y + 7z = 2$ and normal to the plane $3x - 2y + 4z = 5$.

8. A sphere of radius k passes through the origin and meets the axes in A, B, C. Prove that the centroid of the triangle ABC lies on the sphere $9(x^2 + y^2 + z^2) = 4k^2$.

9. Show that the equation to the cone whose vertex is (α, β, γ) and the base parabola $z^2 = 4ax,\ y = 0$ is $(\beta z - \gamma y)^2 = 4a(\beta - y)(\beta x - \alpha y)$.

SECTION "E"

[5 Q.× 2 = 10 marks]

10. What does the equation $x^2 - y^2 + 2x + 4y = 0$ become if the origin is transferred to the parallel axes through the point $(1, -2)$?

11. Find the equation of the plane through the points $(-1, 1, -1)$ and $(6, 2, 1)$ normal to the plane $2x + y + z = 5$.

12. Find the point where the lines $x = \frac{y-2}{2} = \frac{z+3}{3}$ and $\frac{x-2}{2} = \frac{y-6}{3} = \frac{z-3}{4}$ intersect.

13. Find the equation of the sphere having the circle, $x^2 + y^2 + z^2 = 9,\ x - 2y + 2z = 5$ as a great circle.

14. Find the equation of the cone with vertex at the origin and which passes through the curve of intersection of $ax^2 + by^2 + cz^2 = 1$ and $\alpha x + \beta y + \gamma z = \delta$.