Marks Scored:

KATHMANDU UNIVERSITY
End Semester Examination
February/March, 2019

Level : B.Sc.
Year : II
Course : MATH 204
Semester: I

Exam Roll No. :
Time: 30 mins.
F. M. : 20

Registration No.:
Date :

SECTION "A"
[10 Q. × 1 = 10 marks]

Fill in the blank space (s) by most appropriate word (s) or symbol (s).

1. The equation $x\cos\alpha + y\sin\alpha = p$ becomes $x = p$ when the axes are turned through the angle _____ where the symbols have their usual meanings.
2. The equation of the chord joining two points $P(r_1, \theta_1)$ and $Q(r_2, \theta_2)$, on the conic $\frac{\ell}{r} = 1 + e\cos\theta$ is _____.
3. The equation of a curve $x^2 + y^2 - 2xy - 2x - 1 = 0$ represents a(n) _____.
4. If a line is a join of the two points $(1, 2, 3)$ and $(2, 3, 4)$, then its projections on the axes are _____.
5. The points $(1, -1, 2)$ and $(3, 2, -1)$ lie on _____ sides of the plane $x + 2y - 3z - 4 = 0$.
6. If the line $\frac{x-2}{2} = \frac{y+3}{5} = \frac{z-5}{n}$ is parallel to the plane $2x - 3y + z = 3$, then $n =$ _____.
7. The equation of the tangent plane at (α, β, γ) of the sphere $x^2 + y^2 + z^2 = a^2$ is_____.
8. The intersection of the sphere and a plane that passes through the centre point of the sphere is called _____circle.
9. The second degree homogeneous equation in x, y and z represents a cone with its vertex at _____.
10. The locus of tangent lines drawn from a given point to a given surface is known as the_____ cone of that surface with the given point as vertex.

SECTION "B"

[10 Q. × 1 = 10 marks]

Fill in the blank space (s) by choosing the most appropriate answer from among the given ones. **DO NOT TICK** the answers.

11. The coordinate axes should be rotated through the angle _____ so that the expression $x^2 - 4xy + y^2$ may be reduced to the form $x'^2 + y'^2$.

[0; $\frac{\pi}{4}$; $\frac{\pi}{2}$; π]

12. If the axis of the conic $\frac{\ell}{r} = 1 + e\cos\theta$ makes an angle _____with the initial line, the equation of the conic becomes $\frac{\ell}{r} = 1 + e\cos(\theta - \alpha)$.

[α; θ; $\theta - \alpha$; $\alpha + \theta$]

13. The polar of (r', α) with respect to the conic $\frac{\ell}{r} = 1 + ecos\theta$ is _____.

[$\left(\frac{\ell}{r} - e\cos\theta\right)\left(\frac{\ell}{r'} - e\cos\alpha\right) = \cos(\theta - \alpha)$; $\left(\frac{\ell}{r} - e\cos\theta\right)\left(\frac{\ell}{r'} - e\cos\theta\right) = \cos(\theta + \alpha)$;

$\left(\frac{\ell}{r} - e\cos\theta\right)\left(\frac{\ell}{r'} - e\cos\theta\right) = \cos\alpha$; $\left(\frac{\ell}{r} - e\cos\theta\right)\left(\frac{\ell}{r'} - e\cos\theta\right) = e\cos\alpha$]

14. The director circle of any central conic is the locus of points of the tangents drawn from which to the conic are _____.

[Collinear; Parallel; Perpendicular; Non-coplanar]

15. $by + cz + d = 0$ represents the plane parallel to _____.

[x −axis; y − axis; z −axis; $x = a$]

16. The three planes will form a triangular prism if the lines of intersection of the plane taken in pairs are _____.

[Perpendicular; Collinear; Parallel; Intersecting]

17. Two skew lines in three dimensions are _____.

[Parallel; Intersecting; Coplanar; Non coplanar]

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18. The tangent planes to the spheres $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + 4 = 0$ and $x^2 + y^2 + z^2 + 2u_1x + 2v_1y + 2w_1z - 3 = 0$ at any common point are at right angles if $2uu_1 + 2vv_1 + 2ww_1 =$_____.

$[-3;$ $1;$ $4;$ $-12]$

19. The equation to a right circular cone whose vertex is at O; axis OX and semi-vertical angle α is _____.

$[x^2 + y^2 = z^2 \tan^2 \alpha;$ $y^2 + z^2 = x^2 \tan^2 \alpha;$

$z^2 + x^2 = y^2 \tan^2 \alpha;$ $x^2 + y^2 - z^2 = \tan^2 \alpha]$

20. Equation of enveloping cone of the sphere with the vertex at given point is _____.

$[S = T_1;$ $SS_1 = T^2;$ $S^2 = TT_1;$ $SS_1 = TT_1]$

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SECTION "C"

[3 Q. × 7 = 21 marks]

1. Find the equation of the chord of the conic section $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$, whose middle point is at $P(x_1, y_1)$. Find the middle point of the chord $9x - 4y = 14$ of the conic $2x^2 + xy - 3y^2 = 1$. [4+3]

OR

Derive the equation of the chord of a conic $\frac{\ell}{r} = 1 + e\cos\theta$ joining the points whose vectorial angles are θ_1 and θ_2. A circle of given radius passing through the focus S of a given conic, intersects it in A, B, C and D. Show that $SA.SB.SC.SD$ is a constant. [3+4]

2. Define coplanar lines. Find the condition that the lines $\frac{x-x_1}{\ell} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$ and $a_1x + b_1y + c_1z + d_1 = 0 = a_2x + b_2y + c_2z + d_2$ are coplanar. Prove that the lines $x = \frac{y-2}{2} = \frac{z+3}{3}$ and $\frac{x-2}{2} = \frac{y-6}{3} = \frac{z-3}{4}$ are coplanar and find their plane. [1+3+3]

3. A plane passes through a fixed point (a, b, c) and cuts the axes in A, B and C. Prove that the locus of the center of the sphere $OABC$ is $\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 2$. Find the center and radius of the circle $x^2 + y^2 + z^2 + x + y + z = 4,\ x + y + z = 0$. [3+4]

SECTION "D"

[6 Q. × 4 = 24 marks]

4. If the tangent at any point P of a conic meets the directrix in K, prove that KSP is a right angle.

5. Find the lengths of the axes of the conic $8x^2 + 4xy + 5y^2 - 24(x + y) = 0$.

OR

Find the equation of pair of tangents from $(1, 1)$ to the conic $2x^2 + y^2 - 4x + 2y + 2 = 0$.

6. Find the equation of the plane through the intersection of the planes $2x + 3y + 10z = 8$, $2x - 3y + 7z = 2$ and normal to the plane $3x - 2y + 4z = 5$.

7. Find the magnitude of the shortest distance between the lines $\frac{x}{4} = \frac{y+1}{3} = \frac{z-2}{2}$ and $5x - 2y - 3z + 6 = 0 = x - 3y + 2z - 3$.

8. Find the equations to the spheres which pass through the circle $x^2 + y^2 + z^2 = 5$, $x + 2y + 3z = 3$ and touch the plane $4x + 3y = 15$.

9. Obtain the equation of the cylinder which passes through $y^2 = 4ax$, $z = 0$ and whose generators are parallel to the line $x = y = z$.

SECTION "E"

[5 Q. × 2 = 10 marks]

10. What does the equation $(x - a)^2 + (y - b)^2 = c^2$ become when it is transferred to the parallel axes through the point $(a - c, b)$?

11. Find the center of the conic $x^2 - 4xy + y^2 - 2x - 20y - 11 = 0$.

12. Deduce the equation of the plane through $(2, -3, 1)$ and normal to the line joining the points $(0, 0, 0)$ and $(1, 2, 3)$.

13. Find the equation of the sphere which passes through the point $(0, 0, 0)$ and cuts the axes at $(\alpha, 0, 0)$, $(0, \beta, 0)$and $(0, 0, \gamma)$.

14. Find the equation of the cone whose vertex is at origin and which passes through the curve of intersection of the plane $\ell x + my + nz = p$ and the surface $ax^2 + by^2 + cz^2 = 1$.