Marks Scored:

KATHMANDU UNIVERSITY
End Semester Examination
February/March, 2018

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|---|---|---|
| Level : B.Sc. | | Course : MATH 204 |
| Year : II | | Semester: I |
| Exam Roll No. : | Time: 30 mins. | F. M. : 20 |
| Registration No.: | | Date MAR 1 9 2018 |

## SECTION "A"

[10 Q. × 1 = 10 marks]

Fill in the blank space (s) by most appropriate word (s) or symbol (s).

1. If the origin is transferred to $(\alpha, \beta)$ and the axes are tuned through an angle $\theta$, then the transformation will be given by the relations $x =$ ______________, $y =$ ______________.

2. When the axis of the conic $\frac{\ell}{r} = 1 + e\cos\theta$ makes an angle $\pi$ with the initial line, the equation of the conic becomes______________.

3. Centre of the conic $9x^2 - 4xy + 6y^2 - 14x - 8y + 1 = 0$ is at ______________.

4. If the planes $2x + 3y + 4z = 7$ and $2x - my + 5z = 9$ are perpendicular, then $m =$ ___.

5. Where does the line $\frac{x+1}{1} = \frac{y+2}{2} = \frac{z-3}{3}$ intersect the plane $x + y + z = 0$?______________.

6. The angle between the line and the plane is defined as ______________ of the angle between the given line and the line normal to the given plane.

7. The line $\frac{x-x_1}{\ell} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$ lies in the plane $ax + by + cz + d = 0$ if $ax_1 + by_1 + cz_1 + d = 0$ and ______________.

8. The section of a sphere made by a plane passing through the centre of the sphere is called ______________ circle.

9. The equation of the tangent plane at $(\alpha, \beta, \gamma)$ on the sphere $x^2 + y^2 + z^2 = a^2$ is ______________.

10. The locus of all the lines through the vertex at right angles to the tangent planes of the cone is called the ______________ of the given cone.

## SECTION "B"

[10 Q.× 1 = 10 marks]

Fill in the blank space (s) by choosing the most appropriate answer from among the given ones. **Do not tick** the answers.

11. The coordinate axes should be rotated through the angle ________ so that the expression $x^2 - 4xy + y^2$ may be reduced to the form $x'^2 + y'^2$.
[0, $\frac{\pi}{4}$, $\frac{\pi}{2}$, $\pi$]

12. If the two tangents drawn the points $(r_1, \theta_1)$ and $(r_2, \theta_2)$ on the conic $\frac{\ell}{r} = 1 + e\cos\theta$ intersect at the point $(r', \alpha)$, then $r' =$ ____________________ and $\alpha = \frac{\theta_2+\theta_1}{2}$.

[$\frac{\ell}{\cos\frac{\theta_2-\theta_1}{2}+e\cos\frac{\theta_2+\theta_1}{2}}$, $\frac{\ell}{\cos\frac{\theta_2+\theta_1}{2}+e\cos\frac{\theta_2-\theta_1}{2}}$, $\frac{\ell}{\cos\frac{\theta_2+\theta_1}{2}-e\cos\frac{\theta_2-\theta_1}{2}}$, $\frac{\ell}{\cos\frac{\theta_2-\theta_1}{2}-e\cos\frac{\theta_2+\theta_1}{2}}$]

13. The equation of chord of a conic $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ whose middle point is at $P(x_1, y_1)$ is given by ____________________.
[$T = S$, $T = S_1$, $T_1 = S$, $T_1 = S_1$]

14. The equation $by + cz + d = 0$ represents a plane that is parallel to the line ________________.
[$x$ –axis, $y$ –axis, $z$ – axis, $y = x$]

15. Three planes intersect in a point if __________________, where the symbols have their usual meanings.
[$\Delta_1 = \Delta_2 = \Delta_3$ , $\Delta_4 \neq 0$, $\Delta_4 = 0$ but $\Delta_3 \neq 0$ $\Delta_1 = \Delta_3 = 0$]

16. The two lines $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ and $\frac{x-1}{2} = \frac{y-3}{p} = \frac{z-4}{10}$ are coplanar if $p =$ __________________.
[3, 5, 8, 10]

17. Skew lines are ________________________.
[Intersecting, Coincident, Parallel, Non-coplanar]

18. If $S_1 = 0$ and $S_2 = 0$ are two spheres then $S_1 - S_2 = 0$ represents ______________________.
[Circle, Cylinder, Sphere, Straight Line]

19. The plane $\ell x + my + nz = p$ touches the sphere $x^2 + y^2 + z^2 = a^2$ if $p^2 =$ ______________.
[$a^2$, $a^2\ell^2$, $a(\ell^2 + m^2)$, $a^2(\ell^2 + m^2 + n^2)$]

20. The surface generated by a straight line which passes through a fixed point and intersects a given curve is called ______________________.
[Cone, Cylinder, Envelope, Circle]

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SECTION "C"
[3 Q.× 7 = 21 marks]

1. Define directrix of a conic section. Derive the equation of the tangent at the point whose vectorial angle is $\theta_1$ of the conic $\frac{\ell}{r} = 1 + e\cos\theta$. If a chord $PQ$ of a conic whose eccentricity $e$ and semi latus rectum is $\ell$ subtends a right angle at the focus $S$. Prove that $\left(\frac{1}{SP} - \frac{1}{\ell}\right)^2 + \left(\frac{1}{SQ} - \frac{1}{\ell}\right)^2 = \left(\frac{e}{\ell}\right)^2$. [1+3+3]

2. What are coplanar lines? Derive the condition that the lines $\frac{x-x_1}{\ell} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$ and $a_1x + b_1y + c_1z + d_1 = 0 = a_2x + b_2y + c_2z + d_2$ become coplanar. Show that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ are coplanar, find the equation of the plane on which they lie. [1+3+3]

**OR**

What is a line of shortest distance? Find the shortest distance between the lines $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$ and $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$. Find the equation of the line of the shortest distance. [1+3+3]

3. Find the equation of the sphere when the end points of its diameter are given as $(x_1, y_1, z_1)$ and $(x_2, y_2, z_2)$. A sphere of radius $k$ passes through the origin and meets the axes at $A, B, C$. Prove that the centroid of the triangle $ABC$ lies on the sphere $9(x^2 + y^2 + z^2) = 4k^2$. [3+4]

SECTION "D"
[6 Q.× 4 = 24 marks]

4. Find the angle through which the axes must be rotated to remove the term containing $xy$ in $3x^2 + 2xy + 3y^2 - \sqrt{2}\,x = 0$. Also, find the transformed equation.

5. Prove that in any conic, the semi-latus rectum is a harmonic mean between the segments of any focal chord.

6. What conic does the equation $14x^2 - 4xy + 11y^2 - 44x - 58y + 71 = 0$ represent? Find the centre of the given conic.

OR

Show that the joint equation of the axes of the central conic $ax^2 + 2hxy + by^2 = 1$ is $h(x^2 - y^2) = (a - b)xy$.

7. Find the middle point of the chord $9x - 4y = 14$ of the conic $2x^2 + xy - 3y^2 = 1$.

8. Find the equation of the plane which is perpendicular to the plane $5x + 3y + 6z + 8 = 0$ and which contains the line of intersection of the planes
$x + 2y + 3z - 4 = 0,\ 2x + y - z + 5 = 0.$

9. Find the equation of the sphere having the circle $x^2 + y^2 + z^2 = 9,\ x - 2y + 2z = 5$ as a great circle; determine its centre and radius.

## SECTION "E"

[5 Q.× 2 = 10 marks]

10. Transform the equation $x^2 + 2cxy + y^2 = a^2$ by turning the rectangular axes through the angle $\frac{\pi}{4}$.

11. Determine the value of $\lambda$ for which the equation $2x^2 - y^2 - \lambda z^2 + xy + 3yz = 0$ represents a pair of planes?

12. Find where the line $\frac{x-1}{2} = \frac{y+2}{-3} = \frac{z+3}{4}$ meet the plane $2x + 4y - z + 1 = 0$.

13. Find the equation of the tangent plane to the sphere $x^2 + y^2 + z^2 - 2x - 4y + 2z - 3 = 0$ at the point $(-1, 4, -2)$.

14. Show that the line $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$ where $2l^2 + 3m^2 - 5n^2 = 0$ is a generator of the cone $2x^2 + 3y^2 - 5z^2 = 0$.