

KATHMANDU UNIVERSITY
End Semester Examination [C]
December, 2024

Marks Scored:

Level : B.Sc.
Year : II

Course : MATH 204
Semester : I

Exam Roll No. :

Time: 30 mins.

F. M. : 20

Registration No.:

Date 19 DEC 2024

SECTION "A"

[10 Q. \times 1 = 10 marks]

Fill in the blank space(s) by most appropriate words or symbol(s):

1. If the origin of the rectangular coordinates is transferred to a point (h, k) , the transformation will be given by $x = \underline{\hspace{2cm}}$, $y = \underline{\hspace{2cm}}$.
2. $\frac{\ell}{r} = e \cos\theta$ is the equation of the $\underline{\hspace{2cm}}$ of the conic $\frac{\ell}{r} = 1 + e \cos\theta$.
3. The equation $x^2 - 2xy + y^2 - 2x - 1 = 0$ represents a(an) $\underline{\hspace{2cm}}$.
4. The direction cosines of the line equally inclined to the axes are $\underline{\hspace{2cm}}$.
5. The length of the perpendicular from a point $(0, 0, 0)$ to the plane $x - 2y + 2z + 3 = 0$ is $\underline{\hspace{2cm}}$.
6. The equation (in symmetrical form) of a straight line passing through two given points (x_1, y_1, z_1) and (x_2, y_2, z_2) is $\underline{\hspace{2cm}}$.
7. The perpendicular distance between two non-intersecting lines (non-parallel) lines is known as the line of $\underline{\hspace{2cm}}$.
8. The center of the sphere, $x^2 + y^2 + z^2 - 2x - 4y - 6z + 10 = 0$ is at $\underline{\hspace{2cm}}$.
9. The section of a sphere made by a plane passing through the center of the sphere is known as a $\underline{\hspace{2cm}}$.
10. Every homogenous equation of the second degree in x, y, z represents a (an) $\underline{\hspace{2cm}}$ whose vertex is at the origin.

SECTION "B"

[10 Q. \times 1 = 10 marks]

Fill in the blank space(s), **DO NOT TICK**, by selecting the most appropriate answers from among the given ones.

11. The xy term from the expression $ax^2 + 2hxy + by^2$ can be removed by rotating the coordinate axes through the angle, θ given by $\tan 2\theta =$ _____.
- $\left[\frac{h}{a-b}, \quad \frac{2h}{a-b}, \quad \frac{h}{2(a-b)}, \quad \frac{a-b}{2h} \right]$
12. The equation of the chord of contact of the tangents drawn from the point (r', α) is _____.
- $\left[\left(\frac{\ell}{r} - e \cos \theta \right) \left(\frac{\ell}{r'} - e \cos \alpha \right) = \cos(\theta - \alpha), \quad \frac{\ell}{r} = \cos(\theta - \alpha) + e \cos \theta, \right.$
 $\left. \frac{\ell}{r} = e \cos(\theta - \alpha), \quad \frac{\ell}{r} = 1 + e \cos(\theta - \alpha) \right]$
13. The locus of the middle points of a system of parallel chords of a conic is called _____.
- [Diameter, Polar line, Transverse line, Tangent]
14. The center of the conic, $x^2 - 4xy + y^2 - 2x - 20y - 11 = 0$ is at _____.
- [(0, 0), (-7, -4), (-7, 4), (7, -4)]
15. If the projections of a line on the axes are 6, 2, 3, then the length of the line is _____.
- [6, 2, 3, 7]
16. The equations to the straight lines through (a, b, c) parallel to the x -axis are _____.
- $[x = a, y = b, \quad x = b, y = c, \quad y = b, z = c, \quad z = a, x = b]$
17. For what value of k , the equation $2x^2 - y^2 - kz^2 + xy + 3yz = 0$ represents a pair of perpendicular planes? _____.
- [-2, -1, 1, 2]
18. The point where the two straight lines $\frac{x}{1} = \frac{y-2}{2} = \frac{z+3}{3}$ and $\frac{x-2}{2} = \frac{y-6}{3} = \frac{z-3}{4}$ intersect is _____.
- [(2, 6, 3), (0, 2, -3), (2, 3, 6), (1, 2, 3)]
19. The equation of the tangent plane to the sphere $x^2 + y^2 + z^2 = 9$ at $(1, -2, 2)$ is _____.
- $[x - 2y = 9, \quad x - 2y + 2z = 9, \quad x - 2y + 2z = 4, \quad xyz = -4]$
20. A surface generated by a variable straight line that is parallel to a fixed line is called _____.
- [Cylinder, Cone, Circle, Guiding Curve]

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F. M. : 55

SECTION "C"

[3 Q. × 7 = 21 marks]

1. What is conic? Derive the equation of conic in polar form. If a chord PQ of a conic whose eccentricity is e and semi-latus rectum is ℓ subtends a right angle at the focus S , prove that: $\left(\frac{1}{SP} - \frac{1}{\ell}\right)^2 + \left(\frac{1}{SQ} - \frac{1}{\ell}\right)^2 = \left(\frac{e}{\ell}\right)^2$. [1+3+3]
2. What are coplanar lines? Derive the condition that the lines $\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$ and $a_1x + b_1y + c_1z = 0 = a_2x + b_2y + c_2z$ become coplanar. Show that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ are coplanar. [1+3+3]

OR

What is a line of shortest distance? Find the shortest distance between the lines

$\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$ and $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$. Find the equation of the line of the shortest distance. [1+3+3]

3. Define tangent plane. Find the tangent plane equation at any point (α, β, γ) of a sphere. A sphere of radius k passes through the origin and meets the axes in A, B and C . Prove that the centroid of the triangle ABC lies on the sphere $9(x^2 + y^2 + z^2) = 4k^2$. [1+3+3]

SECTION "D"

[6 Q. × 4 = 24 marks]

4. Translate the axes to change the equation $3x^2 - 2xy + 4y^2 + 8x - 10y + 8 = 0$ into an equation with linear term missing.
5. Find the condition that the line $\frac{\ell}{r} = A \cos \theta + B \sin \theta$ may touch the conic $\frac{\ell}{r} = 1 + e \cos \theta$.

P.T.O.

6. Show that $(a^2 + b^2)(x^2 + y^2) = (bx + ay - ab)^2$ is a parabola. Also, prove that its latus rectum is $\frac{2ab}{\sqrt{a^2 + b^2}}$.

OR

Prove that the lengths of the semi-axes of the conic $ax^2 + 2hxy + ay^2 = d$ are $\sqrt{\frac{d}{a+h}}$ and $\sqrt{\frac{d}{a-h}}$ respectively and that their equations are $x^2 - y^2 = 0$.

7. Find the intercepts made on the coordinate axes by the plane $x + 2y - 2z = 9$ and find the direction cosines of the line normal to this plane.
8. Find the equation of the sphere having the circle $x^2 + y^2 + z^2 = 9$, $x - 2y + 2z = 5$ as a great circle. Determine its center and radius.
9. Show that the equation to the cone whose vertex is (α, β, γ) and the base parabola $z^2 = 4ax$, $y = 0$ is $(\beta z - \gamma y)^2 = 4a(\beta - \gamma)(\beta x - \alpha y)$.

SECTION "E"

[5 Q. \times 2 = 10 marks]

10. If the axes are turned through the angle $\frac{\pi}{4}$, what does the equation $xy = 2$ become?
11. Find the polar of the point $(2, 3)$ concerning the conic $x^2 + 4xy - y^2 + 2x - 4y + 5 = 0$.
12. Find the value of k so that the line $\frac{x-2}{2} = \frac{y+3}{5} = \frac{z-5}{k}$ is parallel to the plane $2x - 3y + z = 3$.
13. Find the equation of the sphere described on the join of $(3, 4, 5)$ and $(1, 2, 3)$ as diameter.
14. Find the vertex of the cone, $2x^2 + 2y^2 + 7z^2 - 10yz - 10zx + 2x + 2y + 26z - 17 = 0$.