

KATHMANDU UNIVERSITY
End Semester Examination [C]
November/December, 2023

Marks Scored:

Level : B.Sc.

Year : II

Course : MATH 204

Semester : I

Exam Roll No.:

Time: 30 mins.

F. M. : 20

Registration No.:

Date

05 DEC 2023

SECTION "A"

[10Q. × 1 = 10 marks]

Fill in the blank space(s) by the most appropriate word(s) or symbol(s).

1. The transformation in which directions of the axes are changed without changing the origin is known as _____.
2. When the axis of the conic $\frac{\rho}{r} = 1 + e \cos \theta$ makes an angle α with an initial line, the equation of the conic becomes _____.
3. The general equation of second degree $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents a hyperbola if $\Delta \neq 0$ and _____.
4. Two lines such that the pole of each line with respect to a conic lies on the other line are called _____ lines.
5. If α, β, γ be the angles which a line makes with the co-ordinate axes, then $\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma - 2 =$ _____.
6. If the planes $2x + 3y + 4z = 7$ and $2x - my + 5z = 9$ are parallel, then $m =$ _____.
7. Lines which are not parallel and which do not intersect at a point are called _____.
8. The equation of second degree $mx^2 + my^2 + mz^2 + 2ax + 2by + 2cz + d = 0, m \neq 0$ represents _____.
9. The spheres $x^2 + y^2 + z^2 + 2u_1x + 2v_1y + 2w_1z + d_1 = 0$ and $x^2 + y^2 + z^2 + 2u_2x + 2v_2y + 2w_2z + d_2 = 0$ intersect orthogonally if _____.
10. The locus of all the lines through the vertex at right angles to the tangent planes of the cone is called _____ of the given cone.

SECTION "B"

[10 Q. × 1 = 10 marks]

Fill in the blank space (s) by choosing the most appropriate answers from among the given ones.
Do not tick the answers.

11. For the removal of the xy term from $ax^2 + 2hxy + ay^2$, the rectangular axes must be turned through the angle _____.
[π ; $\frac{\pi}{2}$; $\frac{\pi}{4}$; 0]

12. $\frac{l}{r} = 1 + 2\cos\theta$ is the equation of _____.
 [Hyperbola; Ellipse; Parabola; Circle]
13. The projection of a line on the axes are 3, 2, 6. The length of the line is _____.
 [2; 3; 6; 7]
14. The equation of pair of tangents from the point (x_1, y_1) to the general conic is _____.
 [$SS_1 = T^2$; $TS_1 = S^2$; $S_1 = T^2$; $T = S_1$]
15. The equation $by + cz + d = 0$ represents a plane that is parallel to the line _____.
 [x -axis; y -axis; z -axis; $y = x$]
16. Direction cosines of a line having the projections 6, 2, 3 on the co-ordinate axes are _____.
 [6, 2, 3; 12, 4, 6; $\frac{6}{7}, \frac{2}{7}, \frac{3}{7}$; $\frac{6}{49}, \frac{2}{49}, \frac{3}{49}$]
17. If the line $\frac{x-x_1}{l} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$ is normal to the plane $ax + by + cz + d = 0$, then _____.
 [$\frac{a}{l} = \frac{b}{m} = \frac{c}{n}$; $\frac{a}{x_1} = \frac{b}{y_1} = \frac{c}{z_1}$; $al + bm + cn = 0$; $ax_1 + by_1 + cz_1 = 0$]
18. The line $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12}$ intersects the plane $x - y + z = 5$ at _____.
 [(2, -1, 2); (4, 0, -1); (0, -2, -3); (1, -1, -1)]
19. The plane $\ell x + my + nz = p$ touches the sphere $x^2 + y^2 + z^2 = a^2$ if $p =$ _____.
 [a ; $\sqrt{\ell^2 + m^2 + n^2}$; $a\sqrt{\ell^2 + m^2 + n^2}$; $a^2\sqrt{\ell^2 + m^2 + n^2}$]
20. The right circular cone whose vertex is at O ; axis OX and semi-vertical angle α , is _____.
 [$y^2 + z^2 = x^2 \tan^2 \alpha$; $z^2 + x^2 = y^2 \tan^2 \alpha$;
 $x^2 + y^2 = z^2 \tan^2 \alpha$; $x^2 + y^2 + z^2 = \tan^2 \alpha$]

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Year : II
Time : 2 hrs. 30 mins.

Course : MATH 204
Semester : I
F. M. : 55

SECTION "C"

[3Q. × 7 = 21 marks]

1. Define chord of contact of conic section. Find the equation of the chord of contact of (r', α) with respect to the conic $\frac{\ell}{r} = 1 + e \cos \theta$. Let PSP' and QSQ' be two perpendicular focal chords of a conic $\frac{\ell}{r} = 1 + e \cos \theta$; prove that $\frac{1}{PP'} + \frac{1}{QQ'}$ is constant, S being the focus of the conic. [1+3+3]
2. How is the angle between a plane and a straight line defined? Derive the condition for the general equation of second degree to represent a pair of planes. Find the intercepts made on the coordinate axes by the plane $x + 2y - 2z = 9$ and find the direction cosines of the line normal to this plane. [1+3+3]

OR

Define skew lines and the line of shortest distance. Find the shortest distance between the lines $\frac{x-3}{2} = \frac{y-4}{3} = \frac{z-5}{4}$ and $\frac{x-4}{3} = \frac{y-5}{4} = \frac{z-7}{5}$. Also, find the equation of the shortest distance. [2+3+2]

3. Define sphere. Derive the equation of sphere when two ends of its diameter are given. Find the equations of the sphere through the circle $x^2 + y^2 + z^2 = 5$, $x + 2y + 3z = 3$ and touching the plane $4x + 3y = 15$. [1+3+3]

SECTION "D"

[6 Q. × 4 = 24 marks]

4. What does the equation $x^2 + 2\sqrt{3}xy - y^2 = 2a^2$ become when the axes are turned through an angle 30° to the original axes?
5. What is focal chord? If PSP' and QSQ' be any two focal chords of a conic which are at right angles to one another, prove that $\frac{1}{SP \cdot SP'} + \frac{1}{SQ \cdot SQ'}$ is constant.
6. What conic does the equation $x^2 - 4xy + y^2 - 2x - 20y - 11 = 0$ represent? Find the centre of the given conic.

OR

Extract the middle point of the chord $9x - 4y = 14$ of the conic $2x^2 + xy - 3y^2 = 1$.

7. Find the equation of the plane through the intersection of the planes $2x + 3y + 10z = 8$ and $2x - 3y + 7z = 2$ and normal to the plane $3x - 2y + 4z = 5$.
8. Find the centre and radius of the circle in which the sphere $x^2 + y^2 + z^2 - 8x + 4y + 8z - 45 = 0$ is cut by the plane $x - 2y + 2z = 3$.

9. Define the reciprocal cone and right circular cylinder. Find the equation of the cone with vertex at (α, β, γ) and base $y^2 = 4ax, z = 0$.

SECTION "E"

[5 Q. \times 2 = 10 marks]

10. Transform the equation $x^2 - y^2 + 2x - 3y = 0$ to parallel axes through the point $(2, -3)$.
11. Show that the equation $6x^2 + 4y^2 - 10z^2 + 3yz + 4zx - 11xy = 0$ represents a pair of perpendicular planes, find their equations.
12. Find where the line $\frac{x-1}{2} = \frac{y+2}{-3} = \frac{z+3}{4}$ meet the plane $2x + 4y - z + 1 = 0$.
13. Find the equation of sphere through the circle $x^2 + y^2 + z^2 = 9, x + 3y + 4z = 2$ and origin $(0, 0, 0)$.
14. Obtain the equation of the cylinder which passes through $y^2 = 4ax, z = 0$ and whose generators are parallel to the line $x = y = z$.