KATHMANDU UNIVERSITY
End Semester Examination [C]
November, 2018

Level : B.Sc.
Year : II

Course : MATH 201
Semester: I

Exam. Roll No.: Time: 30 mins. F.M. : 20

Registration No.: Date : NOV 18 2018

SECTION "A"
[10 Q.×1=10 marks]

Fill in the blank space(s) by the most appropriate answer(s):

1. If (x,y,z) and (ρ,ϕ,θ) be respectively the Cartesian and Spherical coordinates of a point in space then $z=$ ______________________.

2. $\int_1^2\int_y^{y^2} dxdy=$ ______________________.

3. $\int_C (x-y+z-2)ds=$ ______________________, when C is the straight line joining the points from $(0,1,1)$ to $(1,0,1)$.

4. $\lim_{x\to 0+}\frac{1}{1+e^{1/x}}=$ ______________________.

5. If $f(x,y)=\sin^2(3x^2+xy)$ then $\frac{\partial f}{\partial y}=$ ______________________.

6. A function $f(x,y)=x^{1/3}y^{-4/3}\tan^{-1}(y/x)$ is a homogeneous function of degree ______________________.

7. If f is continuous in the polar region R described by $r_1(\theta)\le r\le r_2(\theta)$ $(r_1(\theta),r_1(\theta)\ge 0)$, $\alpha\le\theta\le\beta$, then $\iint_R f(r,\theta)dA=$ ______________________.

8. The ______________________ of an object described by the curve C with density $\delta(x,y,z)$ is $\int_C \delta(x,y,z)\,ds$.

9. If $x=r\cos\theta, y=r\sin\theta$ then $\frac{\partial(x,y)}{\partial(r,\theta)}\times\frac{\partial(r,\theta)}{\partial(x,y)}=$ ______________________, where the symbols have their usual meanings.

10. The integration by parts formula of Stieltjes integral states that if $f(x)\in\uparrow, \alpha(x)\in C$ in $a\le x\le b$, then $\int_a^b f(x)d\alpha(x)+\int_a^b \alpha(x)df(x)=$ ______________________.

SECTION "B"

[10 Q.×1=10 marks]

Fill in the blank space(s), DO NOT TICK, by choosing the most appropriate answers from among the given ones.

11. If $f(x,y)=\begin{cases}\frac{x-y}{x+y}, x\neq y\\ 1,\ x=y\end{cases}$, then $\lim_{y\to 0}\left[\lim_{x\to 0} f(x,y)\right]=$ ________________.

[-1; 0; 1; ∞]

12. Let $f(x)\in C$ in $a\le x\le b$, $\alpha(a)=\lambda$, $\alpha(x)=\lambda+h$ in $a<x\le b$. Then $\int_a^b f(x)\,d\alpha(x)=$ ________________.

[h; $f(a)$; $\alpha(a)h$; $f(a)h$]

13. Let the transformation $u=f(x,y), v=g(x,y)$ with Jacobian J have an inverse with Jacobian j Then ________________________________.

[$Jj=0$; $Jj=-1$; $Jj=1$; $Jj=\pi$]

14. If u(x, y), y = g(x, y), then $\frac{\partial u_{x,y}}{\partial x}$ = ________________________ where symbols have usual meanings.

[f_1; f_2; g_1; g_2]

15. The integral of the form $\int_a^b f(x)\,d\alpha(x)$ is called ________________________ integral.

[Riemann; Stietljes; Lebesgue; Laplace]

16. The function $f(x,y)=|x|(1+y)$ is ________________________ at $(0,0)$.

[not continuous; continuous; differentiable; differentiable but not continuous]

17. $L^{-1}\left\{\frac{1}{s(s-1)}\right\}=$ ________________________.

[$t-e^t$; e^t-t; $t-e^{-t}$; $e^{-t}-t$]

18. If $F(x,y)=0$ and y is an explicit function of x, then $\frac{dy}{dx}=$ ________________________.

[$\frac{F_1}{F_2}$; $\frac{F_2}{F_1}$; $-\frac{F_1}{F_2}$; $-\frac{F_2}{F_1}$]

19. The double integral $\iint_R f(x,y)\,dA$ is equivalent to the integral ________________________, where R is the triangle in the *xy*-plane bounded by the x-axis, the line $y=x$, and the line $x=1$.

[$\int_0^1\int_0^x f(x,y)\,dydx$; $\int_0^1\int_0^x f(x,y)\,dxdy$; $\int_0^1\int_0^y f(x,y)\,dydx$; $\int_0^1\int_0^y f(x,y)\,dxdy$]

20. The spherical coordinates for $x^2+y^2+(z-1)^2=1$ is $\rho=$ ________________________.

[$\sin\phi$; $\cos\phi$; $2\sin\phi$; $2\cos\phi$]

KATHMANDU UNIVERSITY
End Semester Examination [C]
November, 2018

NOV 18 2018

Level : B.Sc.
Year : II
Time : 2 hrs. 30 mins.

Course : MATH 201
Semester: I
F.M. : 55

SECTION "C"
[3 Q.×7=21 marks]

1. What do you mean by a line integral? Describe two applications of a line integral. Find the center of mass of a thin wire lying along the curve $\vec{r}(t) = t\,\vec{i} + 2t\,\vec{j} + \frac{2}{3}t^{3/2}\,\vec{k}$ if the density is $\delta = \sqrt{5+t}$. [1 + 2 + 4]

OR

State Stoke's theorem. Verify Stoke's theorem for the field $\vec{F} = x^2\,\vec{i} + 2x\,\vec{j} + z^2\,\vec{k}$ around the curve
C : The ellipse $4x^2 + y^2 = 4$ in the xy – plane, counterclockwise when viewed from above. [2 + 5]

2. Define directional derivative $\frac{\partial f}{\partial \xi_\alpha}$ of a function $f(x,y)$ in the direction of ξ_α at the point (a,b). If $f(x,y) \in C^1$, the prove that
$$\frac{\partial f}{\partial \xi_\alpha} = f_1(x,y)\cos\alpha + f_2(x,y)\sin\alpha$$
Consider $f(x,y) = x^2 - 2y$, then show that $\frac{\partial^2 f}{\partial \xi_\alpha^2} = 2\cos^2\alpha$. [1 + 3 + 3]

3. State conditions for relative minimum and maximum for a function of three variables $f(x,y,z)$ at a point (a,b,c). Show that $f(x,y,z) = (x+y+z)^3 - 3(x+y+z) - 24xyz + a^3$ has a minimum at (1,1,1) and a maximum at $(-1,-1,-1)$. [2 + 5]

SECTION "D"
[6 Q.×4=24 marks]

4. Evaluate $\lim_{x\to 0}\left(\frac{\tan x}{x}\right)^{1/x^2}$.

5. State and prove linearity property of Laplace transform. Use this property to find the Laplace transform of $\sin at$.

6. If $u = 2xy$, $v = x^2 - y^2$, $x = r\cos\theta$ and $y = r\sin\theta$. Eliminate x and y, and thus compute the Jacobian $\frac{\partial(u,v)}{\partial(r,\theta)}$.

OR

Show that $f(x,y) = g(\sqrt{x^2+y^2})$, $g(x) = x^2\sin\frac{1}{x}$, $g(0) = 0$ is differentiable function at (0,0) but not belonging to C^1.

7. Find the Fourier series of $f(x) = x^2, -\pi \le x \le \pi$.

8. Evaluate the Stieltjes integral $\int_0^1 x^2 dx^2$ using definition of Stieltjes integral.

9. Evaluate the integral $\int_0^4 \int_0^1 \int_{2y}^2 \frac{4\cos(x^2)}{2\sqrt{z}} \, dx \, dy \, dz$.

SECTION "E"
[5 Q.×2=10 marks]

10. Find the center and radius of the sphere $= -6\cos\phi$.

11. If $f(x,y) = x\tan^{-1}(x^2+y)$, find $f_2(1,0)$.

12. Evaluate $\int_0^{\pi} x d(\sin x)$.

13. If $u = x^u + u^y$, find $\frac{\partial u}{\partial x}$.

14. Evaluate $\int_C \frac{x^3}{y} \, ds$, where $C: y = \frac{x^2}{2}$, $0 \le x \le 2$.