KATHMANDU UNIVERSITY
End Semester Examination [C]
May/June, 2019

Marks scored:

Level : B.Sc.
Year : II

Course : MATH 201
Semester : I

Exam Roll No. :

Time: 30 mins.

F. M. : 20

Registration No.:

Date : 31 MAY 2019

SECTION "A"
[10 Q. × 1= 10 marks]

Fill in the blank space(s) by the most appropriate answer(s).

1. If $z = \sqrt{x^2 - 2xy - 1}$, then $\frac{\partial z}{\partial x}$ at the point $(1, -2, -2)$ = _____.

2. The center of the sphere $\rho = 1$ is _____, where the symbol has its usual meaning.

3. The area of the planar region $R : 0 \leq x \leq 1,\ 0 \leq y \leq 1$ is _____ square unit.

4. $\lim_{x \to 0} e^{-1/|x|}$ = _____.

5. Definite integral of the form $\int_a^b f(x)\, d\alpha(x)$ is known as _____ integral.

6. The Laplace transform of the function $f(t) = t$ is _____.

7. If $f(x, y) \in C^2, f_1 = f_2 = 0$ at (a, b) and $f_{11}f_{22} - f_{12}^2 < 0$ at (a, b), then the function $f(x, y)$ has a _____ at (a, b).

8. The line integral $\int_C ds$ = _____, where $C : y = x,\ 0 \leq x \leq 2$.

9. If $x = r\cos\theta$ and $y = r\sin\theta$, then the Jacobian $\frac{\partial(x,y)}{\partial(r,\theta)}$ = _____.

10. The degree of the homogeneous function $f(x, y) = x^{1/3}\, y^{4/3}$ is _____.

SECTION "B"
[10 Q. × 1 = 10 marks]

Fill in the blank space(s), DO NOT TICK, by choosing the most appropriate answers from among the given ones.

11. If (x, y, z) and (ρ, ϕ, θ) are respectively the Cartesian and Spherical coordinates representation of a point in space, then z = _____.

[$\rho\sin\theta$; $\rho\cos\theta$; $\rho\sin\phi$; $\rho\cos\phi$]

12. Suppose $\vec{F}$ be a vector field on a domain D, and there exists a differentiable function f such that $\vec{F} = \nabla f$, then $\vec{F}$ is called a _____ vector field.

[conservative; scalar potential; curl; divergence]

13. $\int_0^{\pi} \int_0^{2\pi} \int_0^{3} \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta =$ _____.

[4π; 12π ; 36π ; 108π]

14. The rational function $\frac{x^2+y^2}{x+y}$ is continuous at every point except at the point(s) where _____.

[$x^2 + y^2 = 0$; $y^2 = x^2$; $x + y = 0$; $y = x$]

15. The directional derivative $\frac{\partial f}{\partial \xi_\alpha}$ of the function $f(x, y)$ at the point $P(a, b)$ is given by _____, where $\vec{u}$ is a unit tangent vector.

[$\nabla f(P) \times \vec{u}$; $\vec{u} \times \nabla f(P)$; $\nabla f(P) \cdot \vec{u}$; $\nabla f(p)\, \vec{u}$]

16. The surface area of the sphere $\rho = a$ is given by _____.

[$2\pi a$; $2\pi a^2$; $4\pi a^2$; $4\pi a^3$]

17. The abscissa of convergence S_c of the function $f(t) = 1 + e^{-t}$ is _____.

[-1; 0; 1; ∞]

18. The Green's theorem establishes a relation between line integral in plane and a _____ integral.

[Reimann; Stieltjes; double; triple]

19. A function of m independent variables will have _____ derivatives of order n.

[m^n ; m^{n-1} ; n^m; n^{m-1}]

20. The Stieltjes integral $\int_a^b f(x) d\alpha(x)$ exists if _____, where the symbols have their usual meanings.

[$f(x) \in C$; $\alpha(x) \in C$; $f(x) \in \uparrow$; $f(x) \in C$ and $\alpha(x) \in \uparrow$]

KATHMANDU UNIVERSITY
End Semester Examination [C]
May/June, 2019

31 MAY 2019

Level : B.Sc.
Year : II
Time : 2 hrs. 30 mins.

Course : MATH 201
Semester : I
F. M. : 55

SECTION "C"
[3 Q. × 7 = 21 marks]

1. State and prove Euler's theorem for a homogeneous function $f(x, y)$ of degree n. Verify Euler's theorem for the function $f(x, y) = x^2 + y^2$. [2 + 3 + 2]

OR

State and prove Rolle's theorem. Verify Rolle's theorem for the function $f(x, y) = x^2 + x$ on the interval $[0, 1]$. [2 + 3 + 2]

2. Define a conservative vector field $\vec{F}$. If $\vec{F}(x, y) = M(x, y)\,\vec{\imath} + N(x, y)\,\vec{\jmath}$ is a conservative vector field, then show that $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$. Find the scalar potential function of the conservative vector field $\vec{F}(x, y) = 2xy\,\vec{\imath} + x^2\,\vec{\jmath}$. [1 + 2 + 4]

3. State Fubini's first form for calculating double integrals in a rectangular region $R : a \le x \le b,\ c \le y \le d$. Find the equivalent polar integral of the Cartesian integral

$$\int_{-1}^{1} \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} x^2 + y^2 \, dx\, dy$$

and then evaluate the polar integral. [2 + 5]

SECTION "D"
[6 Q. × 4 = 24 marks]

4. Define linearity property of the Laplace transform, and hence find the Laplace transform of $f(t) = \sin at$.

5. Evaluate the Stieltjes integral of $\int_0^{\pi} \cos x \, d(\sin x)$.

OR

Show that $\int_{-1}^{1} x \, d(|x|) = 1$.

6. Show that $\lim_{x \to a} \frac{\log(x-a)}{\log(e^x - e^a)} = 1$.

7. Find the Spherical equation of the Cartesian equation $x^2 + y^2 + (z-2)^2 = 4$.

8. Find the Fourier series of $f(x) = x,\ -\pi \leq x \leq \pi$.

9. Prove that the function $f(x,y) = \begin{cases} \frac{xy}{\sqrt{x^2+y^2}}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$ is continuous at $(0,0)$.

SECTION "E"

[5 Q. × 2 = 10 marks]

10. Compute f_1 if $f(x,y) = x^y$, where symbol has its usual meaning.

11. Find the Laplace inverse of $\frac{1}{s(s-1)}$.

12. If $x = 4u + 3v,\ y = 3u + 2v$, then find $\frac{\partial u}{\partial x}$.

13. Find the relative maximum value of $f(x) = 1 - x^6$.

14. Use double integral to find the area of the region bounded by the lines $x + y = 1,\ x = 0$, and $y = 0$.

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May/June, 2019

Marks Scored:

Level : B.E.
Year : II

Course : CIEG 204
Semester : I

Exam Roll No.:

Time: 30mins.

F. M. : 10

Registration No.:

Date 31 MAY 2019

SECTION "A"
[20Q. × 0.5 = 10 marks]

Choose the most appropriate answer among the given options and ***encircle*** *the letter of your choice.*

1. If the dynamic viscosity of a fluid is 0.5 poise and specific gravity is 0.5, then the kinematic viscosity of that fluid in stokes is
   a. 0.25 b. 0.5 c. 1.0 d. 2

2. Newton's law of viscosity relates
   a. Intensity of pressure and rate of angular deformation
   b. Shear stress and rate of angular deformation
   c. Shear stress, viscosity and temperature
   d. Viscosity and rate of angular deformation

3. Metacentric height for small angle is the distance between the
   a. Centre of gravity and center of buoyancy
   b. Center of gravity and metacenter
   c. Centre of buoyancy and metacenter
   d. Free surface and center of buoyancy

4. The point in the immersed body through which the resultant pressure of the liquid may be considered to act is known as
   a. Centre of gravity b. Center of buoyancy
   c. Centre of pressure d. Metacenter

5. The continuity equation $\rho_1 v_1 A_1 = \rho_2 v_2 A_2$ is based on the following assumptions regarding flow of fluid
   a. Steady flow b. Uniform flow
   c. Incompressible flow d. Frictionless flow

6. The discharge through a V- notch varies as
   a. $H^{1/2}$ b. $H^{3/2}$ c. $H^{5/2}$ d. $H^{5/4}$

7. The horizontal component of force on a curved surface is equal to the
   a. Product of pressure intensity at its centroid and area
   b. Force on a vertical projection of the curved surface
   c. Weight of liquid vertically above the curved surface
   d. Force on the horizontal projection of the curved surface

8. Stream lines and path lines always coincide in case of
   a. Steady flow b. Laminar flow c. Uniform flow d. Turbulent flow

9. Which of the followings has the highest coefficient of discharge?
   a. Sharp edged orifice b. Venturimeter
   c. Borda's mouth piece d. Cipolleti

10. The shear stress distribution for a fluid flowing in between the parallel plates, both at rest, is
a. Constant over the cross section
b. Parabolic distribution across the section
c. Zero at the mid plane and varies linearly with distance from mid plane
d. Zero at plates and increase linearly to midpoint

11. The discharge of a liquid of kinematic viscosity 4 cm^2/sec through a 8cm diameter pipe is 3200 cm^3/sec. The type of flow is
a. Laminar b. Transitional c. Turbulent d. Unsteady

12. The losses are more in
a. Laminar flow b. Transitional flow c. Turbulent flow d. Critical flow

13. Free surface of a liquid tends to contract to the smallest possible area due to force of
a. Surface tension b. Viscosity c. Friction d. Cohesion

14. The total pressure on a horizontally immersed surface is (where w = specific weight of liquid, A = area of immersed surface, and $\bar{x}$ = depth of C.G of immersed surface from liquid surface)
a. (w.A) b. (w. x) c. (w.A $\bar{x}$) d. (w.A/ $\bar{x}$)

15. The intensity of pressure at any point in a liquid is
a. Directly proportional to the area of the vessel containing liquid
b. Directly proportional to the depth of liquid from free surface
c. Directly proportional to the length of the vessel containing liquid
d. Inversely proportional to the depth of liquid from free surface

16. The theoretical velocity of jet at vena contracta is (where H= head of water at vena contracta)
a. 2gH b. H.$\sqrt{2g}$ c. 2g$\sqrt{H}$ d. $\sqrt{2gH}$

17. The property by virtue of which a liquid opposes relative motion between its different layers is called
a. Surface tension
b. Coefficient of viscosity
c. Viscosity
d. Osmosis

18. A pipe of 0.1m^2 cross sectional area suddenly enlarges to 0.3 m^2 cross sectional area. If the discharge of the pipe is 0.3m^2, head loss is
a. 0.3m b. 0.4m c. 0.1m d. 0.2m

19. A vertically immersed surface is shown in the figure 1. The distance of its center of pressure from the water surface is
a. $\frac{bd^2}{12} + \bar{x}$
b. $\frac{d^2}{12} + \bar{x}$
c. $\frac{b^2}{12} + \bar{x}$
d. $\frac{d^2}{12} + 2\bar{x}$

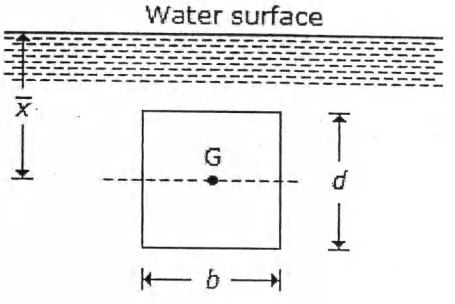

Figure 1

20. The flow in open channel is laminar, if the Reynolds's number is
a. Less than 2000
b. Greater than 2000
c. Greater than 4000
d. Less than 500