KATHMANDU UNIVERSITY
End Semester Examination
March/April, 2025

Marks Scored:

Level : B.Sc.
Year : II

Course : MATH 201
Semester : I

Exam Roll No. :

Time: 30 mins.

F. M. : 20

Registration No.:

Date : 03 APR 2025

SECTION "A"

[10 Q. × 1 = 10 marks]

Fill in the blank space(s) by writing the most appropriate word(s) or symbol(s).

1. The Cartesian coordinates (x, y, z) of the cylindrical coordinates $(r,\ \theta,\ z) = \left(1, \frac{\pi}{2},\ 1\right)$ is ____________________.

2. $\int_0^{2\pi}\int_0^{\frac{\pi}{2}}\int_0^{a} \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta =$ ____________________.

3. If a vector field $\vec{F}(x, y) = M(x, y)\,\vec{i} + N(x, y)\,\vec{j}$ is continuous throughout an open connected region R such that $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$. Then $\oint_C \vec{F} \cdot d\,\vec{r} =$ ____________________, when C is the positively oriented boundary curve of the region R.

4. If $x = r\cos\theta,\ \ y = r\sin\theta$, then the Jacobian $\frac{\partial(x,y)}{\partial(r,\theta)} =$ ____________________.

5. $\lim_{(x,y)\to(0,0)} \frac{xy+4}{x^2+2y^2} =$ ____________________.

6. $\int_0^1 x^2 dx^3 =$ ____________.

7. $\Gamma\left(-\frac{1}{2}\right) =$ ____________________.

8. The function $f(x) = x^2(1 - x)$ has a point of inflection at $x =$ ____________________.

9. The mass of a thin wire lying along the curve $C\colon \vec{r}(t) = t\,\vec{i} + 2t\,\vec{j} + \frac{2}{3}\,t^{3/2}\,\vec{k},\ \ 0 \le t \le 2$ if the density $\delta = \sqrt{5+t}$ is ____________________.

10. $\iint_R (x^2 + y^2)\, dx\, dy =$ ____________________ where R is the region above the x-axis, and within a circle centered at the origin of radius R.

SECTION "B"

[10 Q.× 1 = 10 marks]

Fill in the blank space(s), **DO NOT TICK**, by selecting the most appropriate answers from among the given ones.

11. Suppose (r, θ, z) and (ρ, ϕ, θ) represent respectively the Cylindrical coordinates and Spherical coordinates of a point in space, then ______________________.
[$z = \rho\cos\phi$; $r = \rho\cos\phi$; $z = \rho\cos\theta$; $r = \rho\cos\theta$]

12. $\int_0^1 \int_{x^2}^{x} dy\, dx =$ ______________________.
[0; 1; $\frac{1}{6}$; $\frac{1}{2}$]

13. If a simple closed curve C in the plane and the region R it encloses satisfy the hypotheses of Green's theorem, then the area of the region R is given by ______________________ (counterclockwise).
[$\frac{1}{2}\oint_C y\,dx - x\,dy$; $\frac{1}{2}\oint_C -y\,dx + xdy$; $\oint_C y\,dx - x\,dy$; $\oint_C -y\,dx + x\,dy$]

14. If $u = f(x, y),\ y = g(x, z)$, then $\frac{\partial u_{y,z}}{\partial y} =$ ______________________.
[$\frac{1}{g_1}$; $\frac{1}{f_1 g_1}$; $f_2 + \frac{f_1}{g_1}$; $f_1 + \frac{f_2}{g_1}$]

15. If $f(x, y)$ is a homogeneous function of degree n, the $x\,f_1(x, y) + y\,f_2(x, y) =$ ______________.
[$n\,f$; f; $(n-1)f$; $(n+1)f$]

16. Let $f(x) = x^2$ in $0 \le x \le 1$, $\alpha(x) = \begin{cases} 0, & 0 \le x < \frac{1}{2} \\ 1, & \frac{1}{2} \le x \le 1 \end{cases}$. Then $\int_0^1 f(x)\, d\alpha(x) =$ ______________.
[0; 1; $\frac{1}{2}$; $\frac{1}{4}$]

17. $\Gamma\left(\frac{1}{3}\right)\Gamma\left(\frac{2}{3}\right) =$ ______________________.
[2π; $2\pi/\sqrt{3}$; $\sqrt{3}\,\pi/2$; $\pi/3$]

18. The Fourier series expansion of $f(x) = x^3,\ -1 \le x \le 1$ with periodic continuation has ______________
[only sine terms; only cosine terms; both sine and cosine terms; only sine terms and a non-zero constant]

19. If $f(x, y, z) = 0$, then the value of $\frac{\partial x}{\partial y}\frac{\partial y}{\partial z}\frac{\partial z}{\partial x} =$ ______________________.
[1; -1; 0; ± 1]

20. If $f(x, y) = 0$, then $\frac{dx}{dy} =$ ______________________.
[$-\frac{f_1}{f_2}$; $\frac{f_1}{f_2}$; $-\frac{f_2}{f_1}$; $\frac{f_2}{f_1}$]

KATHMANDU UNIVERSITY
End Semester Examination
March/April, 2025

Level : B.Sc.
Year : II
Time : 2 hrs. 30 mins.

03 APR 2025

Course : MATH 201
Semester : I
F. M. : 55

SECTION "C"
[3 Q. × 7 = 21 marks]

1.

a. Define directional derivative $\frac{\partial f}{\partial \xi_\alpha}$ of $f(x, y)$ in the direction ξ_α. [1]

b. If $f(x, y) \in C^1$, then prove that $\frac{\partial f}{\partial \xi_\alpha} = f_1(x, y) \cos \alpha + f_2(x, y) \sin \alpha$. Also, prove that $\frac{\partial f}{\partial \xi_\alpha} = \nabla f \cdot \vec{u}$
where $\vec{u}$ is the unit tangent vector. [3 + 1]

c. Calculate the derivative of $f(x, y, z) = x^3 - x\,y^2 - z$ at the point $(1, 1, 0)$ in the direction of the vector $2\,\vec{i} - 3\,\vec{j} + 6\,\vec{k}$. [2]

OR

a. If $f(x, y) \in C^1$ in a domain $D \subseteq \mathbb{R}^2$, then prove that $f(x, y)$ is differentiable at every point of D. [4]

b. Suppose $f(x, y) = \begin{cases} \frac{x^3 - y^3}{x^2 + y^2}, & (x, y) \neq (0,0) \\ 0, & (x, y) = (0,0) \end{cases}$. Find $f_1(0,0)$ and $f_2(0,0)$. Is this f differentiable at $(0,0)$? [3]

2.

a. State and prove the tangential form of Green's theorem in a plane. [1+4]

b. Use Green's theorem to find the counterclockwise circulation for the vector field $\vec{F}(x, y) = (x - y)\,\vec{i} + (x + y)\,\vec{j}$ where C: The square bounded by $x = 0$, $x = 1$, $y = 0$, and $y = 1$. [2]

3.

a. Define Stieltjes integral. [1]

b. If $f(x) \in \uparrow$ in $a \leq x \leq b$, and $\alpha(x) \in C$ in $a \leq x \leq b$, then prove that [4]

$$\int_a^b f(x) d\alpha(x) + \int_a^b \alpha(x) df(x) = \alpha(b)\, f(b) - \alpha(a)\, f(a)$$

c. Find the Stieltjes integral $\int_0^1 x\; d\; e^{2x}$. [2]

P.T.O.

SECTION "D"

[6Q. × 4 = 24 marks]

4. Test the function $f(x,y) = x^3 - y^3 + 3x^2 + 3y^2 - 9x$ for relative maxima, relative minima, and saddle points.

5. Sketch the region of integration, reverse the order of integration, and evaluate the integral

$$\int_0^2 \int_0^{4-x^2} \frac{xe^{2y}}{4-y} dy\, dx$$

OR

Convert the integral

$$\int_{-1}^{1} \int_0^{\sqrt{1-y^2}} \int_0^{x} (x^2 + y^2)\, dz\, dx\, dy$$

to an equivalent integral in cylindrical coordinates and evaluate the integral.

6. Verify the limit

$$\lim_{(x,y)\to(0,0)} \frac{x^2 - y^2}{1 + x^2 + y^2} = 0$$

7. Prove that

$$B(m,n) = 2 \int_0^{\pi/2} \sin^{2m-1}\theta\, \cos^{2n-1}\theta \;\; d\theta$$

and also evaluate the integral

$$\int_0^{\pi/2} \sin^5\theta \cos^7\theta\, d\theta$$

8. Find the equation and inequality in the other two coordinates systems of the following:
   a. $\rho = 1,\ 0 \le \phi \le \frac{\pi}{2}$
   b. $\rho = 2\sin\phi\,(\cos\theta - 2\sin\theta)$

9. If $u + v + w = x,\ u^2 + v^2 + w^2 = 2x - 1,\ u^3 + v^3 + w^3 = 3$, find $\frac{\partial u}{\partial x}$ first directly and then by the Jacobian formula.

SECTION "E"

[5Q. × 2 = 10 marks]

10. Find the area of the circle $x^2 + y^2 = a^2$ using the double integral.

11. Find the Fourier sine series of $f(x) = x, \quad -\pi \le x \le \pi$ with period $2\,\pi$.

12. Examine the continuity of the function $f(x,y) = \begin{cases} \frac{xy}{x^2+y^2}, & (x,y) \ne (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$ at the origin.

13. Evaluate the integral $\int_0^\infty e^{-x}\, x^7\, dx$.

14. Verify the Rolle's theorem for the function $f(x) = x^2 - 1,\ -1 \le x \le 1$.