Marks Scored:

KATHMANDU UNIVERSITY
End Semester Examination
March/April, 2017

Level : B. Sc.
Year : II

Course : MATH 201
Semester : I

Exam Roll No. :

Time : 30 mins.

F. M. : 20

Registration No. :

Date : MAR 27 2017

SECTION "A"

[10 Q. × 1 = 10 marks]

Fill in the blank space(s) by the most appropriate answer(s):

1. The value of the Stieltjes integral $\int_0^1 x^2\, dx^2 =$ ________________.

2. The cylindrical coordinates (r,θ,z) of the Cartesian coordinates $(x,y,z)=(1,0,0)$ is ________________.

3. If $f(x,y,z)=x\sin(yz)$, then $f_3(a,1,\pi)=$ ________________.

4. The abscissa of convergence of $L\{e^{at}\}=\dfrac{1}{s-a}$ is ________________.

5. $\lim\limits_{x\to 0}\dfrac{\sin x}{x^3}=$ ________________.

6. The ________________ of an object described by the curve C with density $\delta(x,y,z)$ is $\int_C \delta(x,y,z)\,ds$.

7. $\int_0^3\int_0^2 (4-y^2)\,dydx=$ ________________.

8. The hypothesis $f(x)\in C^1, a\le x\le b$ and $f(a)<f(c), f(b)<f(c), a<c<b$ imply that there exists at least one number $\xi, a<\xi<b$ such that $f(x)\le f(\xi), a\le x\le b$ and $f'(\xi)=$ ________________.

9. The function $f(x,y)=\dfrac{\sin(xy)}{x-y}$ is not continuous at points in a plane which lie along a line ________________.

10. If $\dfrac{\partial f}{\partial \xi_\alpha}=x\cos\alpha+y\sin\alpha$, then $\dfrac{\partial^2 f}{\partial \xi_\alpha^2}=$ ________________ where $\dfrac{\partial f}{\partial \xi_\alpha}$ is the directional derivative of $f(x,y)$ in the direction ξ_α.

SECTION "B"

[10 Q. × 1 = 10 marks]

Fill in the blank space(s), DO NOT TICK, by choosing the most appropriate answers from among the given ones.

11. $\lim\limits_{x\to 0}\dfrac{1}{x}\int_0^x \dfrac{|\sin t|}{t}\,dt=$ ________________.

 [-1; 0; 1; does not exist]

12. Let R be a region enclosed by a simple closed curve C in the plane oriented in counterclockwise. Then the area of the region R is given by the integral ______________________

[$\oint_C xdy - ydx$; $\oint_C ydx - xdy$; $\frac{1}{2}\oint_C xdy - ydx$; $\frac{1}{2}\oint_C ydx - xdy$]

13. Let the transformation $u = f(x, y), v = g(x, y)$ with Jacobian J have an inverse with Jacobian j. Then

[$Jj = 0$; $Jj = -1$; $Jj = 1$; $Jj = \pi$]

14. The average value of $f(x, y) = xy$ over the rectangle $0 \le x \le \pi, 0 \le y \le \pi$ is ________________.

[π^2; $\frac{\pi^2}{2}$; $\frac{\pi^2}{4}$; $\frac{\pi^2}{2}$]

15. The Stieltjes integral $\int_a^b f(x) d\alpha(x)$ exists if ______________________, $a \le x \le b$.

[$f(x) \in C$; $\alpha(x) \in \uparrow$; $\alpha(x) \in \downarrow$; $f(x) \in C$ and $\alpha(x) \in \uparrow$]

16. The value of the Fourier coefficient b_n in the Fourier series $f(x) = a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$ is equal to ______________________ if we take $f(x) = |x|, -\pi \le x \le \pi$.

[-1; 0; 1; π]

17. A function $f(x, y)$ has an absolute maximum at a point (α, β) of a region R if and only if ______________________ for all (x, y) in R.

[$f(\alpha, \beta) > f(x, y)$; $f(\alpha, \beta) \ge f(x, y)$; $f(\alpha, \beta) < f(x, y)$; $f(\alpha, \beta) \le f(x, y)$]

18. If $x = 4u + 3v, y = 3u + 2v$. Then $\frac{\partial u}{\partial x} =$ ______________________.

[-4; $\frac{1}{2}$; 3; $\frac{5}{2}$]

19. The equation of a sphere of radius a centered at origin in spherical coordinate system is ________________ where the symbols have their usual meanings.

[$r = a$; $r = a\cos\theta$; $\rho = a$; $\rho = \cos\phi$]

20. A function $f(x, y)$ is homogeneous of degree n in a region R if and only if, for $f(x, y)$ in R and for every positive number λ, ______________________.

[$f(\lambda x, \lambda y) = \lambda^{1/n} f(x, y)$; $f(\lambda x, \lambda y) = \lambda^n f(x, y)$;

$f(x, y) = y^n f\left(1, \frac{y}{x}\right)$; $f(\lambda x, \lambda y) = x^n f\left(1, \frac{y}{x}\right)$]

KATHMANDU UNIVERSITY
End Semester Examination
March/April, 2017

MAR 2 7 2017

Level : B. Sc.
Year : II
Time : 2 hrs. 30 mins.

Course : MATH 201
Semester : I
F. M. : 55

SECTION "C"

[3 Q. × 7 = 21 marks]

1. Define Stieltjes integral $\int_a^b f(x)d\alpha(x)$ and use this definition to evaluate the Stieltjes integral for $a = 0, b = 2, \alpha(x) = \begin{cases} 0, & 0 \le x \le 1 \\ 1, & 1 < x \le 2 \end{cases}$ and $f(x) = 1, 0 \le x \le 2$. [2 + 5]

OR

Define bounded variation, and evaluate the Stieltjes integral $\int_0^2 f(x)d\alpha(x)$ given that $f(x) \in C, 0 \le x \le 2$ and $\alpha(x) = \begin{cases} 1, & 0 \le x \le 1 \\ 0, & x = 1 \\ 1, & 1 < x \le 2 \end{cases}$ using the Extension result theorem of bounded variation for existence of Stieltjes integral. [2 + 5]

2. State and prove first shifting theorem of the Laplace transform, and use this theorem to evaluate the Laplace transform of $f(t) = e^{at}\cos\omega t$. [4 + 3]

3. If $f(x,y) \in C^1$, then prove that $\frac{\partial f}{\partial \xi_\alpha} = f_1(x,y)\cos\alpha + f_2(x,y)\sin\alpha$ where $\frac{\partial f}{\partial \xi_\alpha}$ is the directional derivative of $f(x,y)$ in the direction ξ_α. Use this relation to evaluate $\left.\frac{\partial f}{\partial \xi_\alpha}\right|_{(1,1)}$ when $f(x,y) = xy + x\log y$. [4 + 3]

SECTION "D"

[6 Q. × 4 = 24 marks]

4. Evaluate $\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^{\frac{1}{x}}$.

OR

Evaluate $\lim_{h\to 0}\frac{f(x+2h) - 2f(x+h) + f(x)}{h^2}$

5. Evaluate $\int_0^2 [x]dx^2$, where $[x]$ is the greatest integer function.

6. Use Green's theorem to evaluate the integral $\oint_C (3ydx + 2xdy)$, where C: The boundary $0 \le x \le \pi, 0 \le y \le \sin x$.

7. Evaluate $\int_0^1\int_0^1\int_0^1 \left(x^2+y^2+z^2\right) dz\, dy\, dx$.

8. If $x^2+y^2+z^2+u^2=1$, $xy-zu=2$. Compute $\frac{\partial z_{xy}}{\partial x}$ where the symbols have their usual meanings.

9. Find the Fourier series of the function $f(x)=x$, $-\pi \le x \le \pi$.

SECTION "E"

[5 Q. × 2 = 10 marks]

10. Use the Laplace transform to solve $y'(t)+a\,y(t)=1, y(0)=0$, where a is a constant.

11. Evaluate $\int_C x\, ds$, C: The path along the parabola $y=x^2$, $0 \le x \le 1$.

12. If $u=x^2+y^2$, $x=r\cos\theta, y=r\sin\theta$. Compute $\frac{\partial u}{\partial r}$.

13. Find the spherical equation of the Cartesian equation $x^2+y^2+(z-3)^2=9$.

14. Find the absolute maximum and absolute minimum of $f(x)=x^4-4x^3+1, -1 \le x \le 1$