Marks Scored:

KATHMANDU UNIVERSITY
End Semester Examination [C]
June, 2018

Level : B. Sc.
Year : II

Course : MATH 201
Semester : I

Exam Roll No. :

Time: 30 mins.

F. M. : 20

Registration No.:

Date : JUN 18 2018

SECTION "A"
[10 Q. × 1 =10 marks]

Fill in the blank space(s) by the most appropriate answer(s):

1. If $x = r^2$ and $y = r\tan\theta$, then the Jacobian $\frac{\partial(x,y)}{\partial(r,\theta)}$ = ____________.

2. If (r,θ,z) and (ρ,ϕ,θ) are respectively the cylindrical and spherical coordinates of a point. Then z = ______________.

3. $\iint_R x^2 y^5 dA$ = ____________, R: $1 \le x \le 2$, $0 \le y \le 1$.

4. The Laplace transform of $f(t) = t^n, n \in \mathbf{N}$ is ____________.

5. If $\alpha(x)$ be continuous and non-decreasing in $a \le x \le b$, then $\int_a^b d\alpha(x)$ = ________.

6. If $f(x,y) = x^{xy}$, then $\frac{\partial f}{\partial y}$ = ______________.

7. $\lim\limits_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$ = ________________ along path y = x.

8. The mass of an object described by the curve C with density $\delta(x, y, z)$ is given by ______________.

9. The directional derivatives of $f(x,y)$ at a point (a,b) in the direction of $\vec{i}$ is __________________ at (a,b).

10. If $f(x,y) = x^2 + y^2$, then grad f = ________________.

SECTION "B"
[10 Q. × 1 = 10 marks]

Fill in the blank space(s), ***DO NOT TICK***, by choosing the most appropriate answers from among the given ones.

11. If $f(x, y) = x\tan^{-1}(x^2+y)$, then $f_1(1, 0)$ = ____________________ where symbols have their usual meanings.
[1; $1 + \pi/4$; 2; $2 + \pi/4$]

12. If $f(x, y) = x^2 - 2y$, then the directional derivative of f in the direction ξ_α, where $\alpha = 3\pi/4$ at (1, 2) is ________________.

[-2; $-2\sqrt{2}$; 2; $2\sqrt{2}$]

13. If $f(x) \in C, \alpha(x) \uparrow$ in $a \le x \le b$, then ________________ exists.

[$\int_a^b f(x)\alpha(x)dx$; $\int_a^b f(x)d\alpha(x)$; $\int_a^b f(x)\alpha'(x)dx$; $\int_a^b f'(x)\alpha(x)dx$]

14. The function $\sqrt{x} - \sqrt{y}$ is a homogeneous function of degree ________________.

[0; 1/2; 1; 2]

15. Suppose $f(x) = x^2 \sin(1/x)$ when $x \neq 0$, and 0 when x = 0. Then $f_+'(0) =$ ________________.

[-1; 0; 1; none of these]

16. A function $f(x, y)$ has an absolute maximum at a point (X, Y) iff ________________.

[$f(x,y) > f(X,Y)$; $f(x,y) \ge f(X,Y)$; $f(X,Y) > f(x,y)$; $f(X,Y) \ge f(x,y)$]

17. The Cartesian integral $\int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} dy\,dx$ is equivalent to the polar integral ________________.

[$\int_0^{2\pi} \int_{-1}^{1} dr\,d\theta$; $\int_0^{2\pi} \int_{-1}^{1} r dr\,d\theta$; $\int_0^{2\pi} \int_0^1 dr\,d\theta$; $\int_0^{2\pi} \int_0^1 r dr\,d\theta$]

18. The function $f(x, y) = |x|(1 + y)$ is ________________ at $(0,0)$.

[not continuous; continuous; differentiable; differentiable but not continuous]

19. If a vector field $\vec{F}(x, y, z)$ is conservative, then $\vec{F} =$ ________________ for some scalar potential function *f*.

[∇f; $\nabla^2 f$; $\nabla \cdot (\nabla f)$; $\nabla \cdot (\nabla^2 f)$]

20. $f \in C^1 \Rightarrow f$ is ________________, where the symbols have their usual meanings.

[not differentiable; differentiable; continuous but not differentiable; continuous only]

KATHMANDU UNIVERSITY
End Semester Examination [C]
June, 2018

JUN 18 2018

Level : B. Sc.
Year : II
Time : 2 hrs.30 mins.

Course : MATH 201
Semester : I
F.M. : 55

SECTION "C"
[3 Q. × 7 = 21 marks]

1. If $f(x) \in \uparrow$ and $\alpha(x) \in C$ in $a \le x \le b$, then prove that

$$\int_a^b f(x)d\alpha(x) + \int_a^b \alpha(x)df(x) = \alpha(b)f(b) - \alpha(a)f(a)$$

Use this result to evaluate $\int_0^{\pi/2} x\, d\sin x$. [5+2]

OR

Define Stietljes integral. Evaluate the Stietljes integral $\int_0^1 x^2 dx^2$ by definition. [2+5]

2. State a conservative vector field. If a vector field $\vec{F}(x,y) = M(x,y)\,\vec{i} + N(x,y)\,\vec{j}$ is conservative, then show that $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$. Determine $\vec{F}(x,y,z) = yz\,\vec{i} + xz\,\vec{j} + xy\,\vec{k}$ is conservative or not. If yes, find the potential function for it. [1+2+4]

3. Consider the double integral $\int_0^2 \int_0^{\sqrt{1-(x-1)^2}} \frac{x+y}{x^2+y^2}\, dydx$ in Cartesian form. Sketch the region of integration. Find the equivalent polar integral and evaluate this polar integral. [3+4]

SECTION "D"
[6 Q. × 4 = 24 marks]

4. Evaluate $\lim_{x\to\pi/2} (\tan x)^{\cos x}$.

5. Use Green's theorem to find the counterclockwise circulation for the field $\vec{F}(x,y) = (y^2 - x^2)\,\vec{i} + (x^2 + y^2)\,\vec{j}$ and the curve C: the triangle bounded by y = 0, x = 3 and y = x.

6. If $f(x,y) = \sqrt{|xy|}$, show that $f(x,y)$ is not differentiable at (0, 0).

7. Use Laplace transform to find the general solution of $y'(t) - y(t) = 0$.

8. If $x = r\cos\theta$, $y = r\sin\theta$, find $\frac{\partial r}{\partial x}$ by use of Jacobian and then using the explicit inverse of the transformation.

9. Show that the function $f(x,y,z) = 2xyz - 4zx - 2yz + x^2 + y^2 + z^2 - 2x - 4y + 4z$ has a minimum at $(1,2,0)$.

OR

Show that the relative minimum of the function $f(x,y) = x^2 + 2xy + 2y^2 + 4x$ occurs at the point (-4, 2) and hence find the minimum value.

SECTION "E"

[5 Q. × 2 = 10 marks]

10. Show that $\int_0^4 x\,d[x] = 10$, where $[x]$ is the greatest integer function.

11. Evaluate the Triple integral $\int_0^a \int_0^b \int_0^c dx\,dy\,dz$.

12. Evaluate $\lim_{x \to 1/2} \frac{\log 2x}{2x-1}$

13. Find the Cartesian equation of the sphere $\rho = 5\cos\phi$.

14. If $u = x^u + u^y$, find $\frac{\partial u}{\partial x}$.