KATHMANDU UNIVERSITY
End Semester Examination
July/August, 2024

Level : B.Sc.
Year : II
Time : 2 hrs. 30mins.

0 2 AUG 2024

Course : MATH 201
Semester : I
F. M. : 55

SECTION "C"
[3 Q. × 7 = 21 marks]

1. a. State and prove a basic mean value theorem for a function $f(x,y)$. [1+3]
b. Verify the basic mean value theorem for the function
$f(x,y) = x^2 + y^2,\ (a,b) = (1,1)$. [3]

OR

a. Let R be a region, $f(x,y) \in C^1,\ (x,y) \in R$ and $f(x,y)$ is a homogeneous function of degree n in R. Then prove that $x\, f_1(x,y) + y\, f_2(x,y) = n\, f(x,y)$. [4]
b. If $u = \sin^{-1}\left(\frac{x^2+y^2}{x+y}\right)$, then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \tan u$. [3]

2. a. Define the Beta function, $B(m,n)$. [1]
b. Prove that $B(m,n) = \frac{n-1}{m+n-1}\, B(m, n-1)$. [3]
c. Evaluate the integral $\int_0^1 x^4\,(1-x)^2\,dx$ using the properties of beta and gamma functions. [3]

3. a. Use the limit definition of double integral to evaluate the integral $\int_0^1 \int_0^1 f(x,y)\,dA$ when $f(x,y) = x$. [4]
b. Use double integral to find the area of the region R bounded by $y = x$ and $y = x^2$ in the first quadrant. [3]

SECTION "D"
[6 Q. × 4 = 24 marks]

4. Use the $\epsilon - \delta$ definition to verify that $\lim\limits_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2+y^2}} = 0$.

OR

Let $f(x,y) = \begin{cases} \frac{x-y}{x+y}, & x \neq -y \\ 1, & x = -y \end{cases}$. Does the limit at (x,y) approaches $(0,0)$ exist? Give a reason for your argument.

5. Show that the differential forms in the integral $\int_{(0,0)}^{(0,1)} \sin y \cos x\, dx + \cos y \sin x\, dy$ is exact, and then evaluate the integral.

6. Find and classify all critical points for the function $f(x,y) = x^4 + y^4 - 4xy$.

P.T.O.

7. Evaluate the Stieltjes integral $\int_0^5 (x^2+1)\, d[x]$ where $[x]$ is "the largest integer less than x".

8. If (x, y, z) and (ρ, ϕ, θ) represent a point in space in Cartesian and Spherical coordinates systems, respectively. Then show that $x = \rho \sin\phi \cos\theta,\ \ y = \rho \sin\phi \sin\theta,$ and $z = \rho \cos\phi$.

9. If $f(x,y) = \begin{cases} \frac{x^6 - 2y^4}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$. Show that $f(x,y)$ is differentiable at $(0,0)$.

SECTION "E"

[5 Q. × 2 = 10 marks]

10. Evaluate the Stieltjes integral $\int_0^1 d\, x^2$.

11. Find the parametric equation of $y = x^2,\ 0 \leq x \leq 2$.

12. Evaluate the double integral $\int_0^3 \int_0^2 (4 - y^2)\, dy\, dx$.

13. If $u = x^{y^z}$, find $\frac{\partial u}{\partial x}$.

14. Write the equation of the cone $z^2 = x^2 + y^2$ in the Spherical coordinate system.

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SECTION "A"
[10 Q. × 1 = 10 marks]

Fill in the blank space(s) by most appropriate words or symbol(s):

1. The degree of the homogeneous function $f(x,y) = x^{\frac{1}{3}} y^{-\frac{4}{3}} \sin^{-1}\left(\frac{x}{y}\right)$ is ____________.

2. The work done by a force field $\vec{F}(x,y) = x\,\vec{i} + y\,\vec{j}$ over the curve $C: \vec{r}(t) = \vec{i} + t\,\vec{j}$, $0 \le t \le 1$ is ______________.

3. If a function $f(x)$, $a \le x \le b$ satisfy the hypotheses of Rolle's theorem, then there exists a number c, $a < c < b$ such that __________________.

4. $\lim_{x \to 0+} e^{-1/x} =$ ____________________.

5. The function $f(x) = 1 - x^4$ has a relative maximum at $x =$____________________.

6. $\int_0^1 x\, d\, e^{ax} =$ ____________________.

7. If $f(x), -\pi \le x \le \pi$ is an even periodic function with period 2π. The value of the Fourier coefficient b_n in the Fourier series $f(x) = a_0 + \sum_0^\infty a_n \cos nx + b_n \sin nx$ is ________________.

8. The value of the integral $\int_0^1 \int_0^2 \int_0^3 dx\, dy\, dz =$ __________________.

9. The Spherical coordinates (ρ, ϕ, θ) of the cylindrical coordinates $(\sqrt{2}, 0, 1)$ is ______________.

10. Assume that $\iint_R f(x,y)\, dA$ exists, then $\iint_R f(x,y) dA \ge 0$ if ________________.

SECTION "B"

[10 Q. × 1 = 10 marks]

Fill in the blank space(s), ***DO NOT TICK,*** *by selecting the most appropriate answers from among the given ones.*

11. If $u = f(x, u)$. Then $\frac{du}{dx} =$ ______________________.
[$\frac{f_2}{1-f_1}$; $\frac{f_2}{1-f_2}$; $\frac{f_1}{1-f_2}$; $\frac{f_1}{1-f_1}$]

12. The statement ____________________ ____is true if $f(x) = x,\ x \geq 0$ and $f(x) = 0, x < 0, -1 < x < 1$.
[$f(x) \in C, f(x) \in C^1$; $f(x) \notin C, f(x) \notin C^1$; $f(x) \notin C, f(x) \in C^1$; $f(x) \in C, f(x) \notin C^1$]

13. If $[x]$ means "the largest integer $\leq x$". Then $[x]$ is continuous on the interval ____________________________.
[$0 \leq x < 1$; $0 \leq x \leq 1$; $0 < x \leq 1$; $0 \leq x \leq 1.5$]

14. The graph of $f(x) = x^5 - x + 2$ has a point of inflection at $x =$ ________________.
[-1; 0; 1; 2]

15. If $\alpha(x)$ is a non-decreasing function in $a \leq x \leq b$, then $\int_a^b d\,\alpha(x) =$ ____________.
[$b - a$; $a - b$; $\alpha(b) - \alpha(a)$; $\alpha(a) - \alpha(b)$]

16. The value of $\frac{\Gamma(5)}{2\,\Gamma(4)\Gamma(3)} =$ _______________________.
[1; 2; 3; 4]

17. Consider an object described by the curve C with density $\rho(x, y)$, then the mass of the object is given by ____________________.
[$\int_C ds$; $\int_C \rho(x,y)dx$; $\int_C \rho(x,y)dy$; $\int_C \rho(x,y)\,ds$]

18. The elementary area dA in polar form is ____________________.
[$dr\,d\theta$; $\frac{1}{2}\,dr\,d\theta$; $r\,dr\,d\theta$; $r^2dr\,d\theta$]

19. The Cartesian equation of the polar equation $r^2 \cos\theta \sin\theta = 4$ is _____________.
[$xy = 1$; $xy = 4$; $y = 4x$; $x = 4y$]

20. Let $m, n > 0$, then $\frac{\Gamma(m)\ \Gamma(n)}{\Gamma(m+n)} =$ ______________________.
[$B(n, m)$; $B(n-1, m)$; $B(n,\ m-1)$; $B(n+1, m+1)$]